University of Camerino – International School of Advanced Studies

PhD program in Computer Science and Mathematics

Academic Year 2020/2021

APPLIED TOPOLOGY

Prof. Riccardo Piergallini

The course aims to provide some basic topological notions and results needed to introduce persistent homology, compute related topological invariants, and exploit it in application contexts.

Program of the course

Metric and topological spaces. Euclidean spaces, balls, cubes, simplexes and their boundary. Metric spaces, isometries. Topological spaces, continuous maps, homeomorphisms, connected components, compactness. Topological manifolds (with boundary), invariance of dimension. Topological equivalence of balls, cubes and simplexes (regular convexes). Deformations and homotopical equivalence, contractible spaces.

Combinatorial topology. Abstract and topological graphs. Abstract and topological simplicial complexes, cubical complexes, cellular complexes. Subdivisions, barycentric subdivision. Simplicial maps, cellular maps, approximation of continuous maps, embedding theorem in Euclidean spaces. Simplicial collapsings and cellular deformations. Smooth manifolds, smooth maps, diffeomorphisms. Simplicial decompositions, PL structures. Morse functions, handle decompositions, decompositions of spheres and tori. Discrete Morse theory.

Homology theory. Simplicial, cubical, cellular and singular homology with coefficients in Z, Q and R, functoriality and homotopical invariance, equivalence theorems. Betti numbers and Euler characteristic. Mayer-Vietoris exact sequence, Künneth formula, homology of spheres and tori. Morse complex, Morse inequalities. Homology of surfaces, homology of manifolds, Poincarè duality. Alexander duality for subcomplexes in Euclidean spaces.

Persistent topology. Approximation of spaces, Vietoris and Cech complexes, regular neighborhoods in Euclidean spaces, alpha-complexes, Delaunay-Voronoi complexes. Filtrations of spaces and complexes, complexes associated to weighted graphs. Persistent homology, persistence diagrams, bar-codes.

Suggested Textbooks

H. Edelsbrunner and J. Harer, *Computational topology – An introduction*, AMS 2010.

H. Edelsbrunner, *A short course in computational geometry and topology*, Springer 2014.

R. Ghrist, *Elementary applied topology*, https://www.math.upenn.edu/∼ghrist/notes.html.

A.J. Zomorodian, *Topology for computing*, Cambridge University Press 2005.