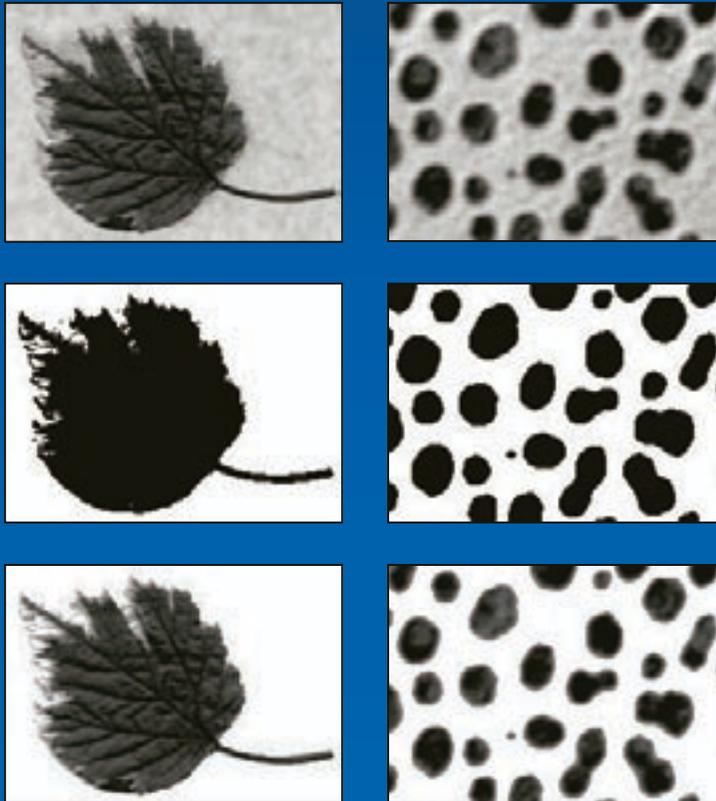
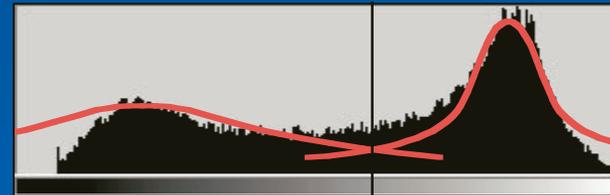


SEGMENTAZIONE

estrazione delle regioni di interesse



soglie globali o locali



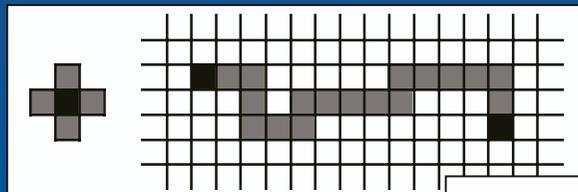
cammini ottimali



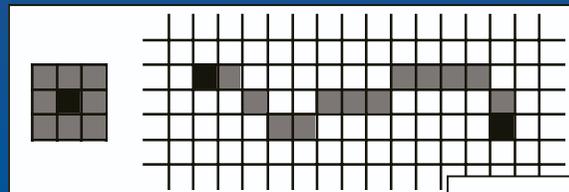
contorni → regioni

$(w(x, y) = 1 - l(x, y))_{x,y}$ (oggetto scuro su sfondo chiaro)

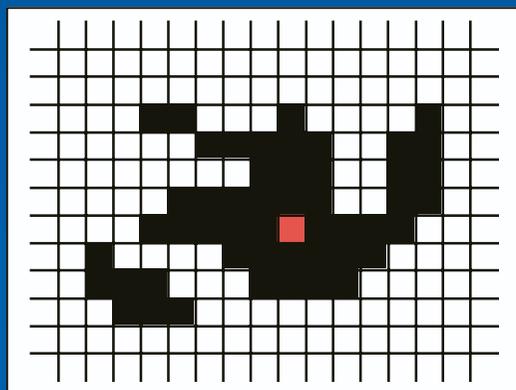
COMPONENTI CONNESSE



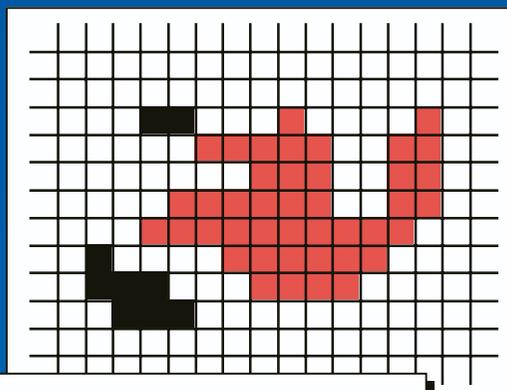
4-cammino



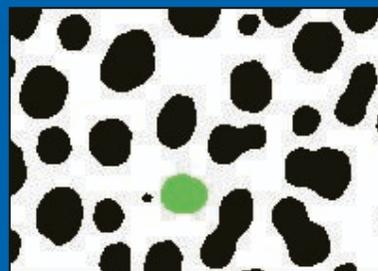
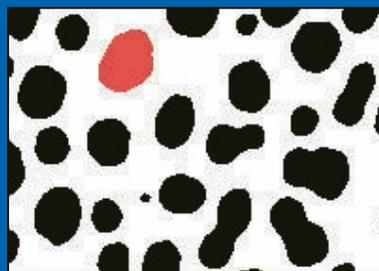
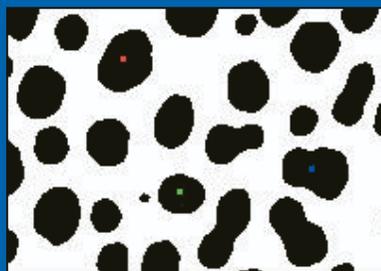
8-cammino



4-componente



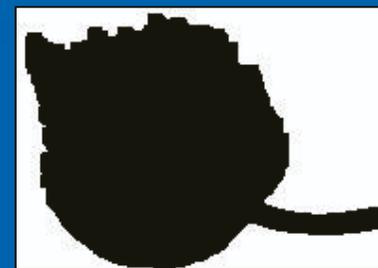
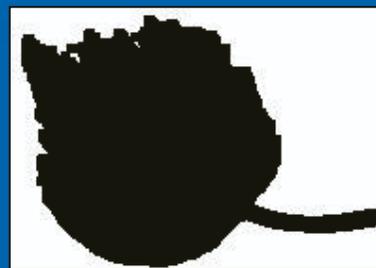
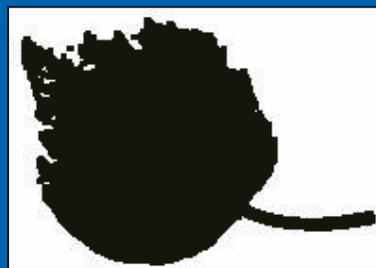
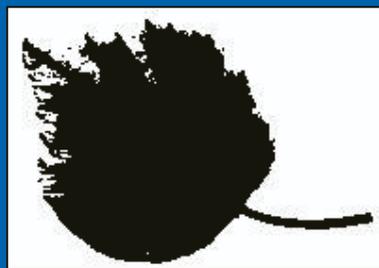
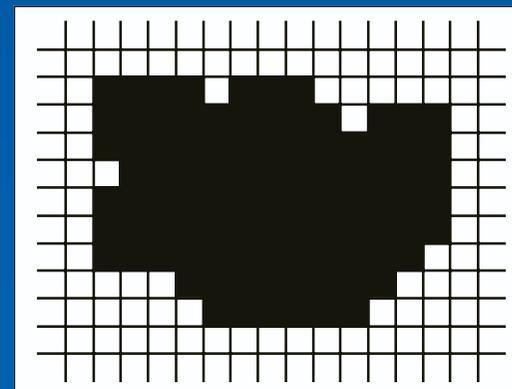
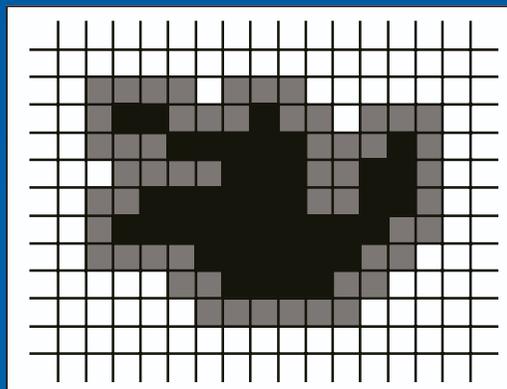
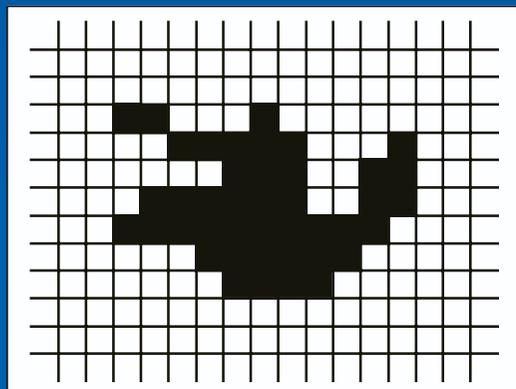
8-componente



DILATAZIONE

$(w(x, y) = 1 - l(x, y))_{x,y}$ (oggetto scuro su sfondo chiaro)

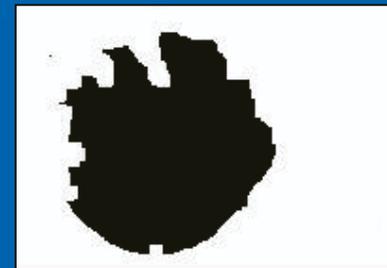
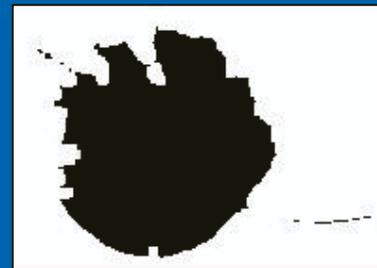
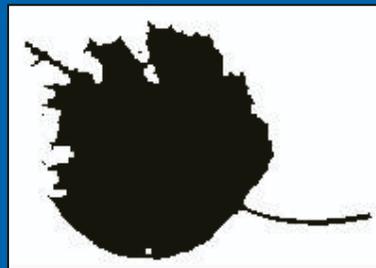
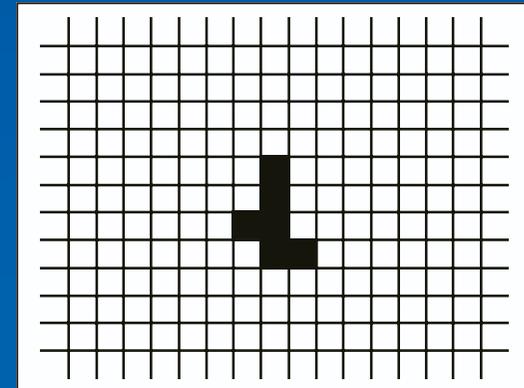
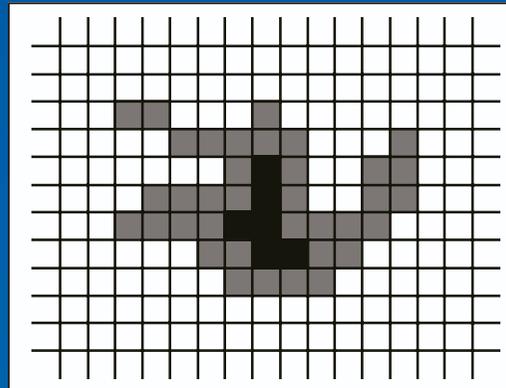
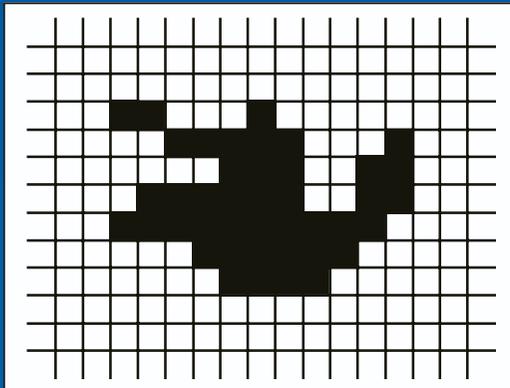
$$w^D(x, y) = \max (w(x + h, y + k))_{B_r/C_r/D_r}$$



EROSIONE

$(w(x, y) = 1 - l(x, y))_{x,y}$ (oggetto scuro su sfondo chiaro)

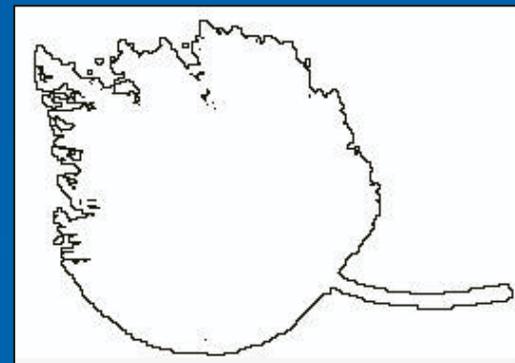
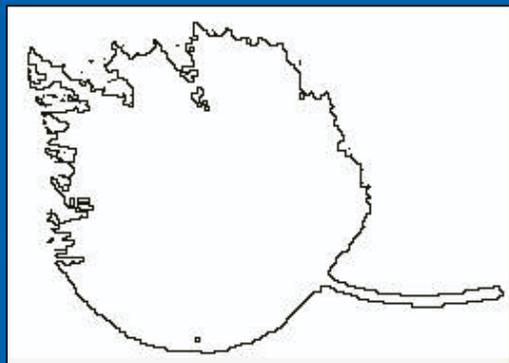
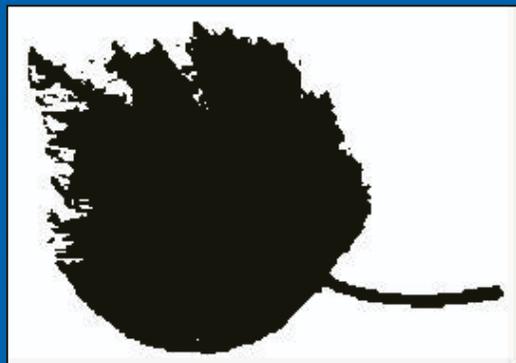
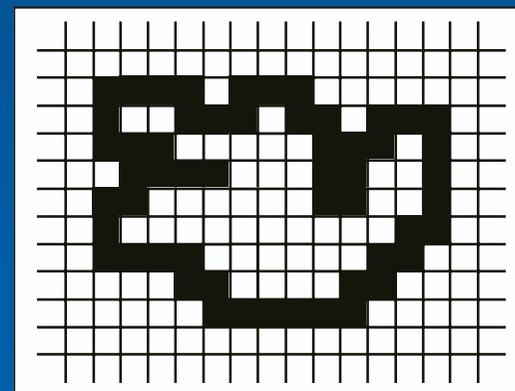
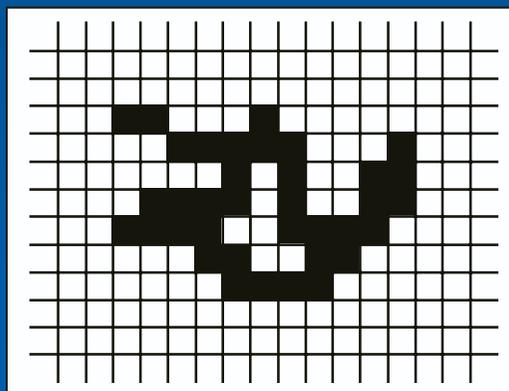
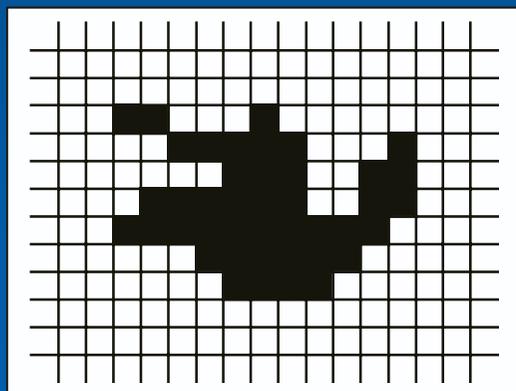
$$w^E(x, y) = \min (w(x + h, y + k))_{B_r/C_r/D_r}$$



CONTORNI

$$w - w^E$$

$$w^D - w$$



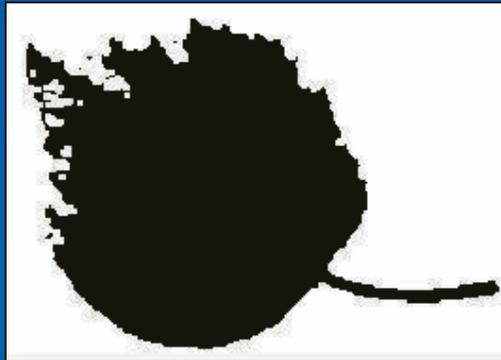
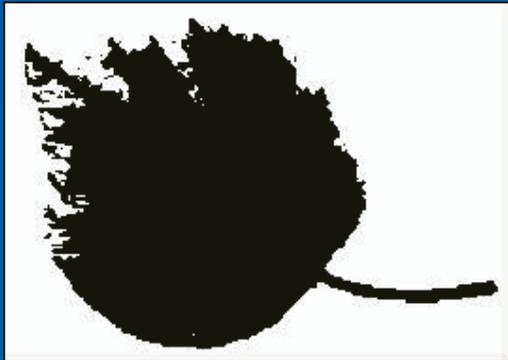
contorno interno

contorno esterno

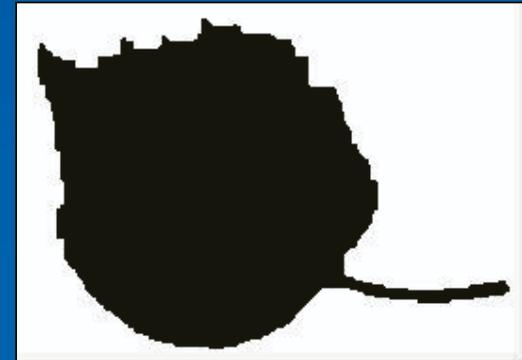
CHIUSURA

$$(w(x, y))_{x,y} \xrightarrow{D} \cdot \xrightarrow{E} (w^C(x, y))_{x,y}$$

$$(w(x, y))_{x,y} \xrightarrow{DD \dots D} \cdot \xrightarrow{EE \dots E} (w^{Cn}(x, y))_{x,y}$$



$n = 1$



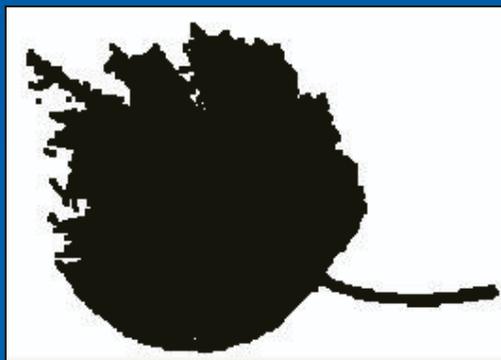
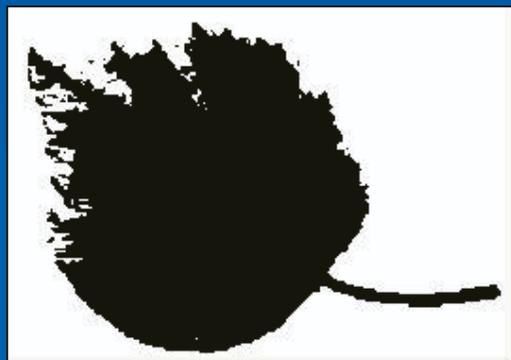
$n = 5$

regolarizzazione verso l'esterno
(eliminazione di "laghi", "stretti" e "golfi" di ampiezza $\leq n$)

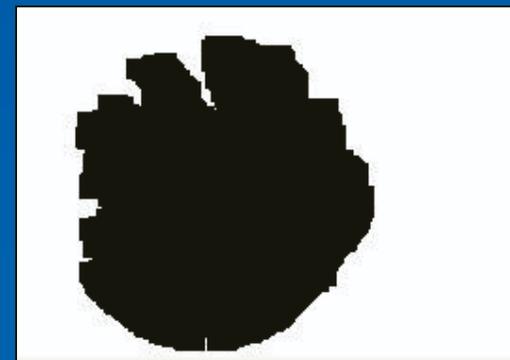
APERTURA

$$(w(x, y))_{x,y} \xrightarrow{E} \cdot \xrightarrow{D} (w^A(x, y))_{x,y}$$

$$(w(x, y))_{x,y} \xrightarrow{E E \dots E} \cdot \xrightarrow{D D \dots D} (w^{An}(x, y))_{x,y}$$



$n = 1$



$n = 5$

regolarizzazione verso l'interno
(eliminazione di "isole", "istmi" e "penisole" di ampiezza $\leq n$)

SMUSSAMENTO

$$w^S(x, y) = \text{mediana}(w(x + h, y + k))_{B_r/C_r/D_r}$$

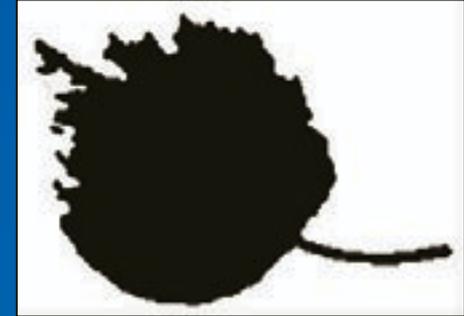
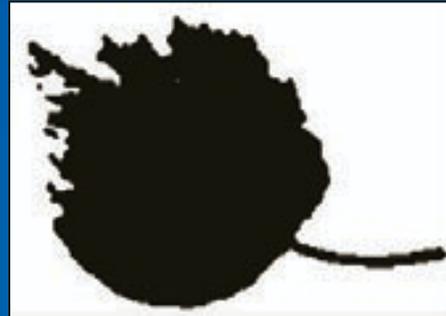
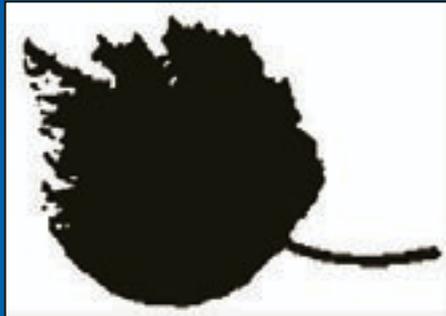
(criterio di maggioranza per immagini binarie)

S

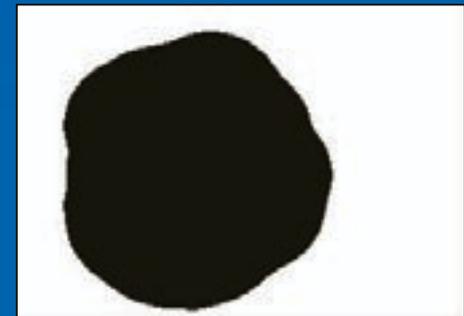
S^5

$S^n = S^{n+1}$

$r = 1$

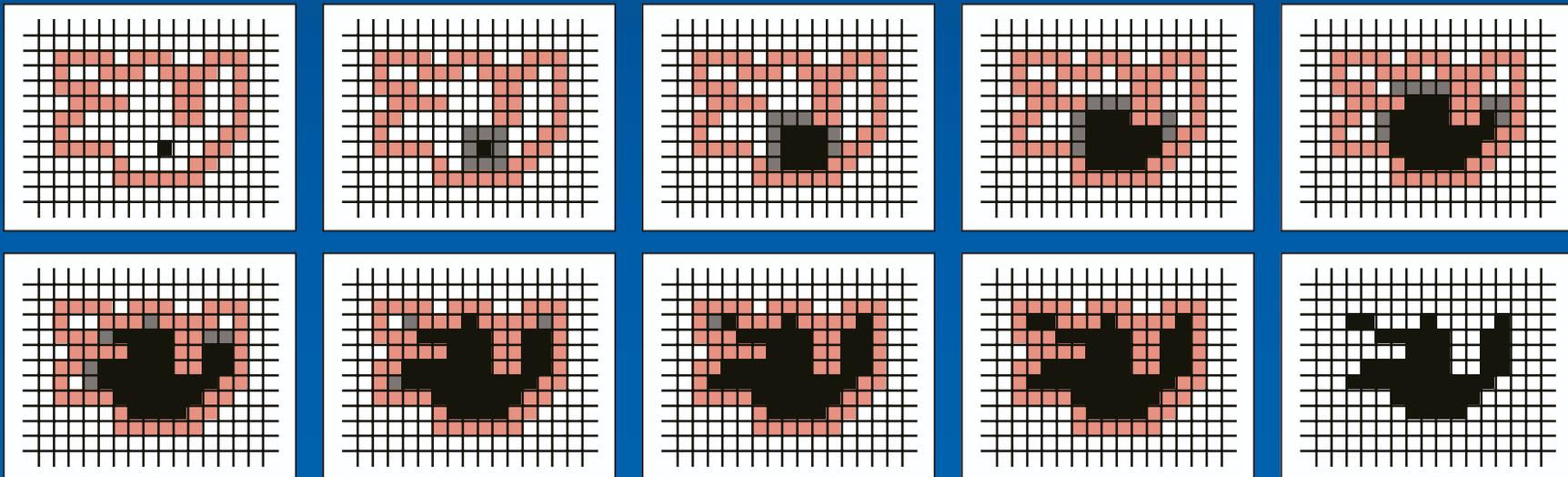


$r = 5$



COMPONENTI

$$p \xrightarrow{D_c D_c \dots D_c} \text{Comp}(p) \quad (D_c = \text{dilatazione condizionata})$$



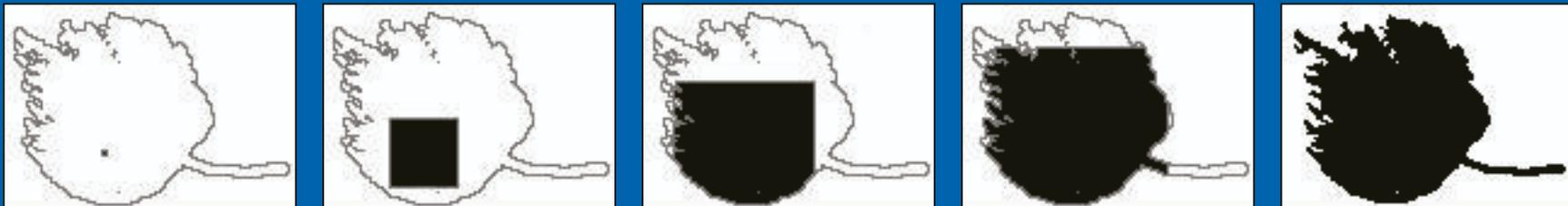
\rightsquigarrow

8-componenti



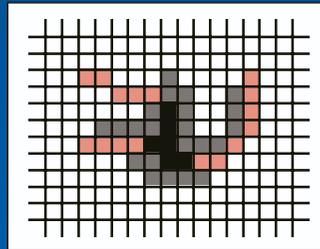
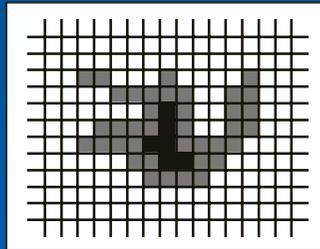
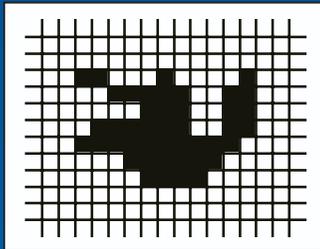
\rightsquigarrow

4-componenti

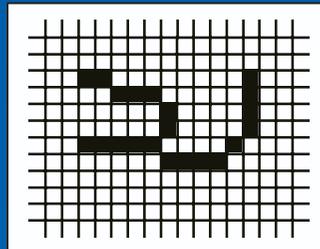
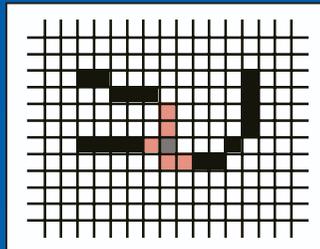
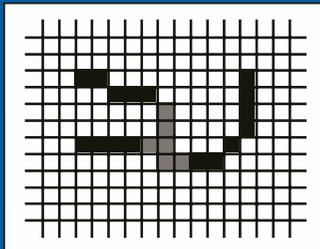


SCHELETRI

$$R \xrightarrow{E_c E_c \dots E_c} \text{Sk}(R) \quad (E_c = \text{erosione condizionata})$$

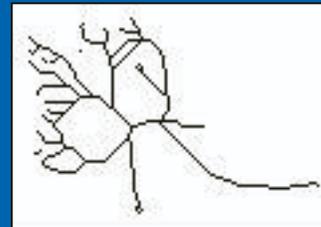
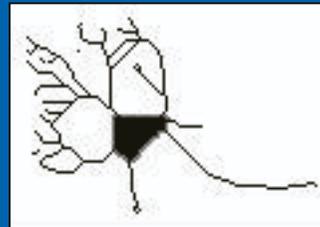
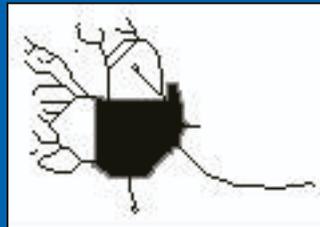
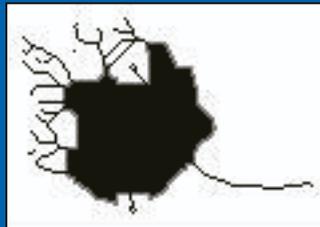


conservazione
delle estremità



conservazione
delle componenti
8-/4-connesse


 \rightsquigarrow 8-scheletro
 
 \rightsquigarrow 4-scheletro

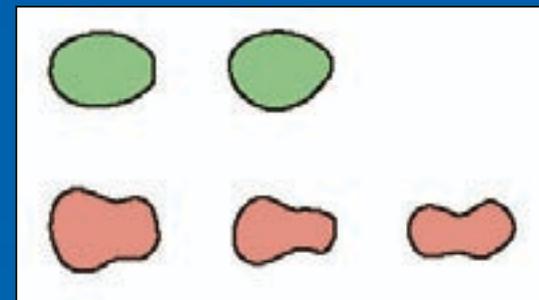
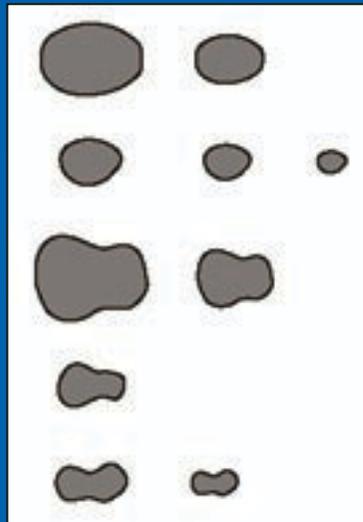
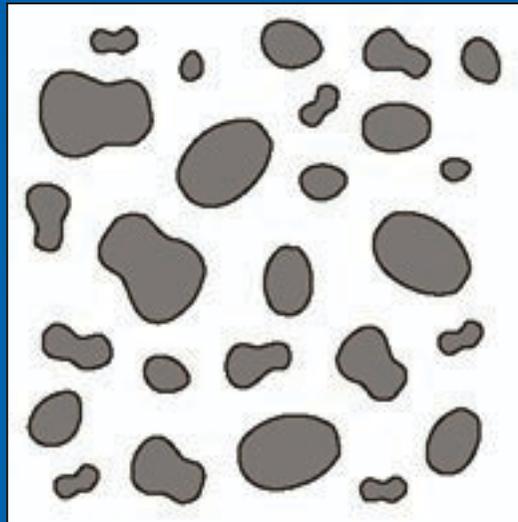


ANALISI MORFOLOGICA

POSIZIONE (baricentro, orientazione)

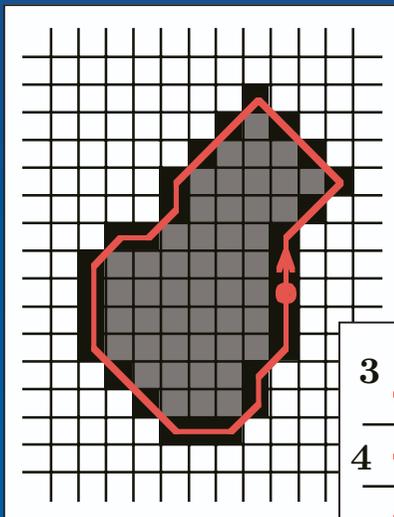
DIMENSIONI indipendenti dalla posizione
invarianti per isometrie (traslazioni, rotazioni, simmetrie)

FORMA indipendente da posizione e dimensioni
invariante per similitudini (isometrie e scalature)



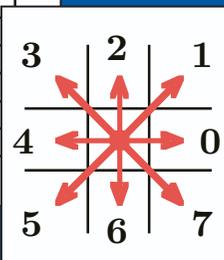
Classificazione

RAPPRESENTAZIONI



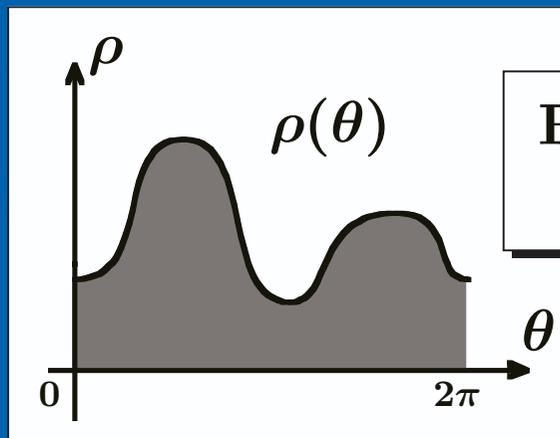
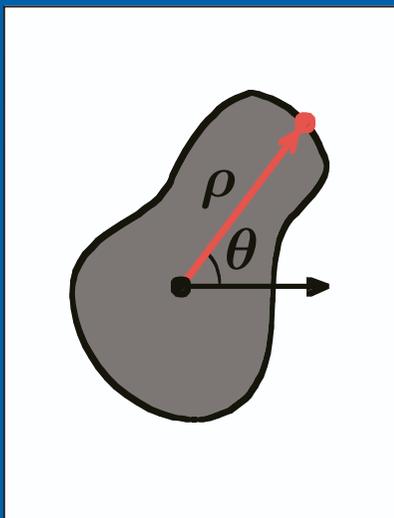
Rappresentazione poligonale

$p_1 = (x_1, y_1), p_2 = (x_2, y_2), \dots, p_n = (x_n, y_n)$
(sequenza ciclica dei pixel del contorno)



Codifica invariante per traslazioni

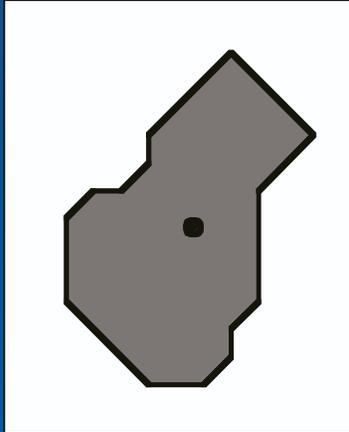
22113335556545666777001212
(sequenza ciclica dei passi elementari)



Rappresentazione polare

($\rho(\theta)$ funzione periodica)

PARAMETRI GEOMETRICI



perimetro $L = \sum_i d(p_i, p_{i+1})$

area $A = \sum_i (x_i y_{i+1} - x_{i+1} y_i) / 2 \simeq N = |R|$

circularità $C = 4\pi A / L^2$ (parametro di forma)

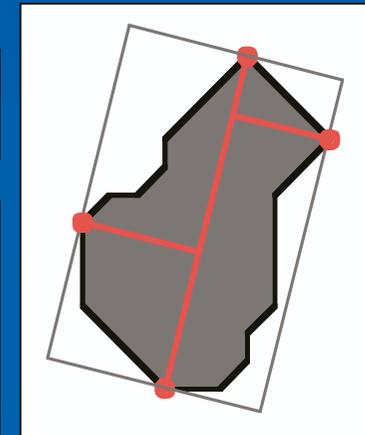
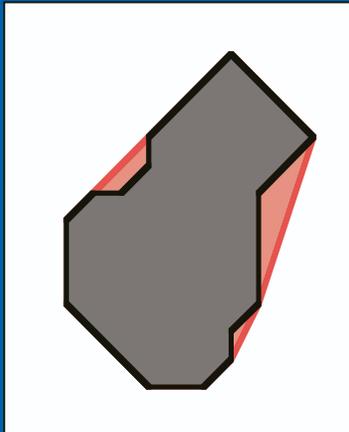
baricentro $\begin{cases} \bar{x} = \sum_R x / N \\ \bar{y} = \sum_R y / N \end{cases}$

diametro $D = \max_{i,j} d(p_i, p_j)$

asse principale ($D =$ lunghezza)

asse secondario ($E =$ larghezza)

D/E (parametro di forma)



confronto tra R e l'involucro convesso $H(R)$

variazione angolare totale $\sum_i |\alpha_i|$ ($\alpha_i = \hat{p}_i$)

PARAMETRI TOPOLOGICI



invarianti per trasformazioni continue

n = numero delle componenti di R

c = numero delle componenti del contorno

b = numero dei buchi ($= c - n$)



invarianti per piccole deformazioni

topologia dello scheletro $Sk(R)$

(numero e ordine dei punti di ramificazione

numero e ordine dei cammini e dei cappi)

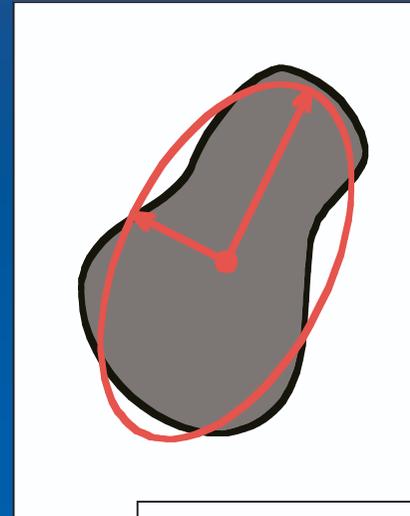
topologia di $H(R) - R$ (numero componenti)

PARAMETRI STATISTICI

direzioni e semiassi principali

autoversori e autovalori $\lambda_1, \lambda_2 > 0$
della matrice di correlazione

$$\begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{xy} & \sigma_{yy} \end{bmatrix} \quad \begin{aligned} \sigma_{xx} &= \Sigma_R (x - \bar{x})^2 / N \\ \sigma_{yy} &= \Sigma_R (y - \bar{y})^2 / N \\ \sigma_{xy} &= \Sigma_R (x - \bar{x})(y - \bar{y}) / N \end{aligned}$$



momenti centrali

$$\mu_{p,q} = \Sigma_R (x - \bar{x})^p (y - \bar{y})^q / N$$

$$\eta_{p,q} = \mu_{p,q} / N^{(p+q)/2} \quad (\eta_{0,0} = 1, \eta_{1,0} = \eta_{0,1} = 0)$$

$$\begin{aligned} S_i &= c \lambda_i / \sqrt{N} \\ \pi S_1 S_2 &\simeq N \end{aligned}$$

parametri di forma

$$\lambda_1 / N, \lambda_2 / N \quad \varphi_1 = \eta_{2,0} + \eta_{0,2} = (\lambda_1 + \lambda_2) / N \geq 1/6$$

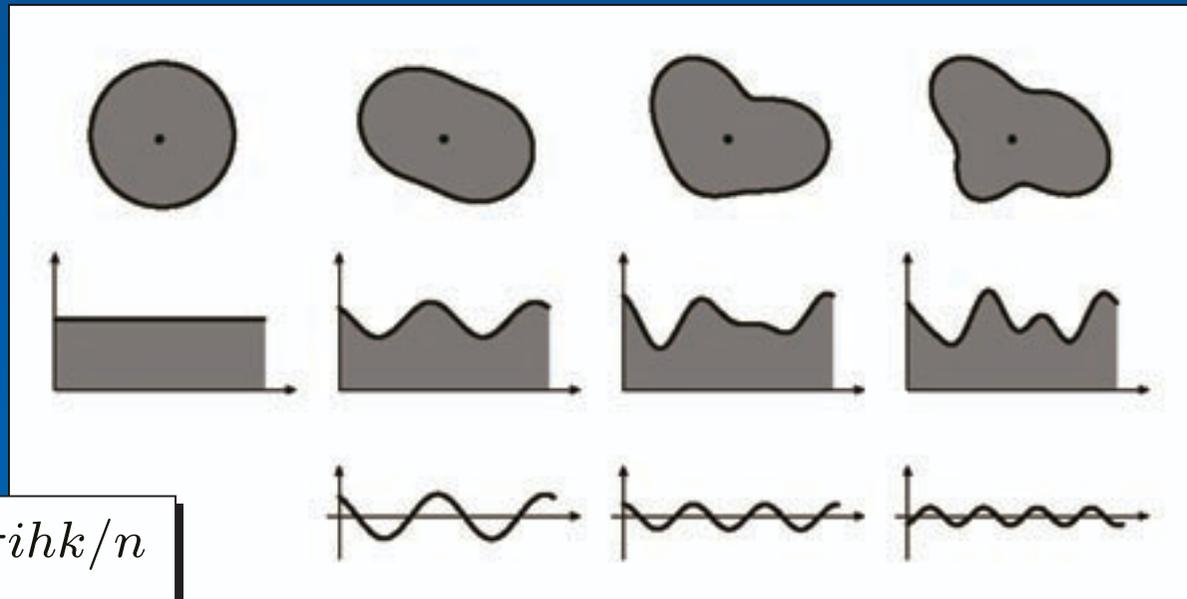
$$R = \lambda_1 / \lambda_2 \quad \varphi_2 = (\eta_{2,0} - \eta_{0,2})^2 + 4\eta_{1,1}^2 \geq 0$$

PARAMETRI DI FOURIER

$$p_0 = (x_0, y_0), p_1 = (x_1, y_1), \dots, p_{n-1} = (x_{n-1}, y_{n-1})$$

$$z_k = x_k + i y_k$$

$$z_k = \sum_{h=0}^{n-1} c_h e^{2\pi i h k / n}$$

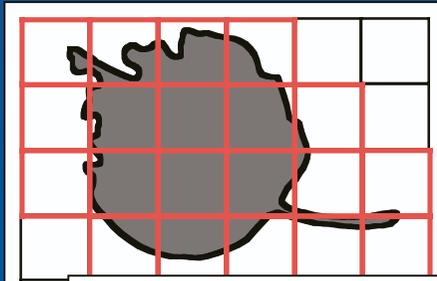


parametri di Fourier

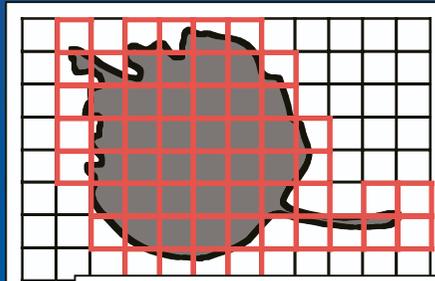
$$c_h = \frac{1}{n} \sum_{k=0}^{n-1} z_k e^{-2\pi i h k / n} \quad h = 0, \dots, n-1$$

$$\rho_h = |c_h| \quad h = 0, \dots, n-1 \quad (\text{parametri di forma})$$

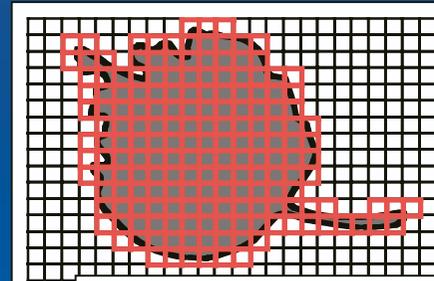
DIMENSIONE FRATTALE



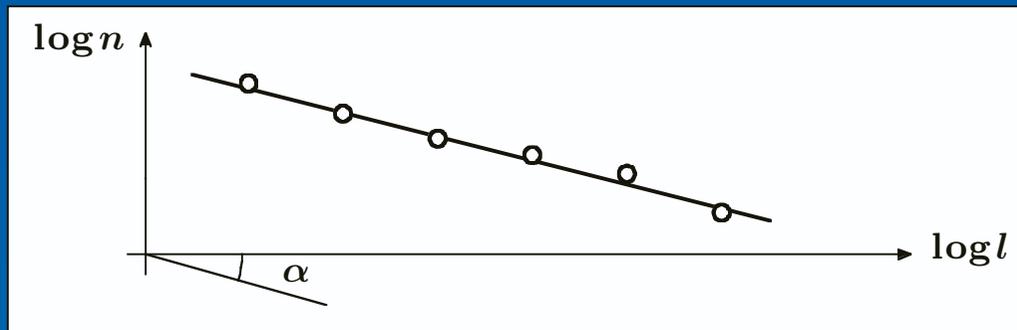
$l = 32 \quad n = 20$



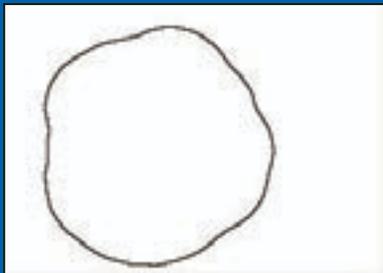
$l = 16 \quad n = 59$



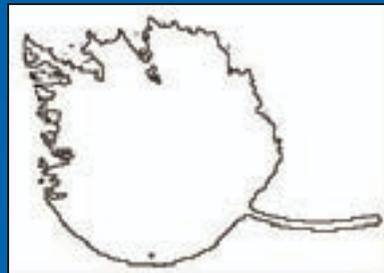
$l = 8 \quad n = 279$



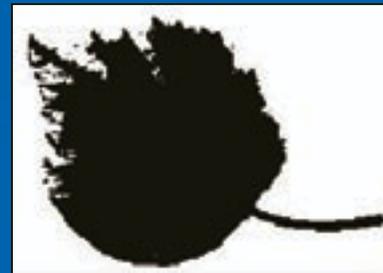
$$D = -\text{tg } \alpha$$



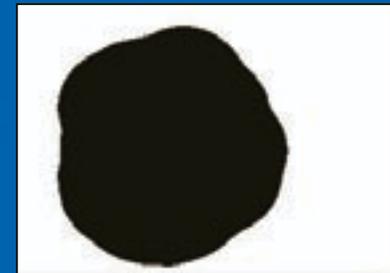
$D = 1.0371$



$D = 1.1196$



$D = 1.8260$



$D = 1.8503$