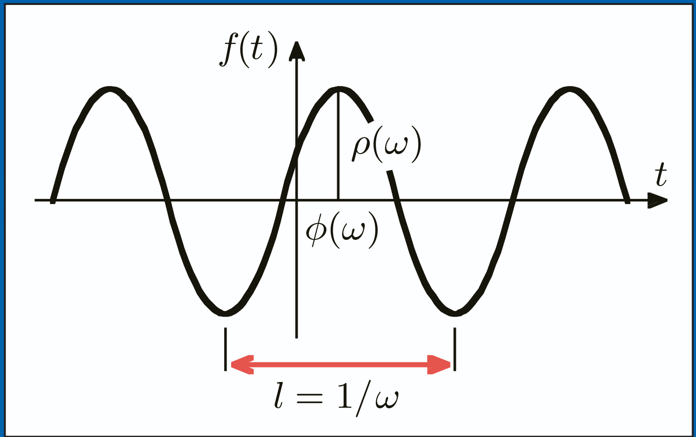
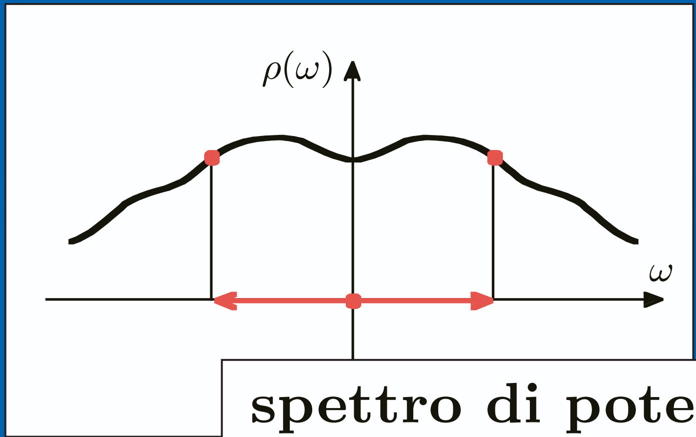
TRASFORMATA DI FOURIER
(una variabile continua)

$$\boldsymbol{f(t) \rightsquigarrow F(\omega) = \int_{-\infty}^{+\infty} f(t) e^{-2\pi i \omega t} dt} \quad (-\infty < t, \omega < +\infty)$$

$$f(t) = \int_{-\infty}^{+\infty} F(\omega) e^{2\pi i \omega t} d\omega = 2 \int_0^{\infty} \rho(\omega) \cos 2\pi(\omega t - \phi(\omega)) d\omega$$

$$F(\omega) = \rho(\omega) e^{-2\pi i \phi(\omega)} \qquad F(-\omega) = \rho(\omega) e^{2\pi i \phi(\omega)}$$





spettro di potenza

TRASFORMATA DI FOURIER
(una variabile discreta)

$$\boldsymbol{f(t) \rightsquigarrow F(\omega) = \frac{1}{N}\sum_{t=1}^{N} f(t)e^{-2\pi i\omega\frac{t}{N}}} \qquad (t,\omega = 1,\ldots,N)$$

$$f(t) = \sum_{\omega=1}^{N} F(\omega)e^{2\pi i\omega\frac{t}{N}} = \sum_{\omega=[-N/2]}^{[N/2]} F(\omega)e^{2\pi i\omega\frac{t}{N}} \qquad (F(\omega \pm N) = F(\omega))$$

$$F(0) = (f(1) + \ldots + f(N))/N$$

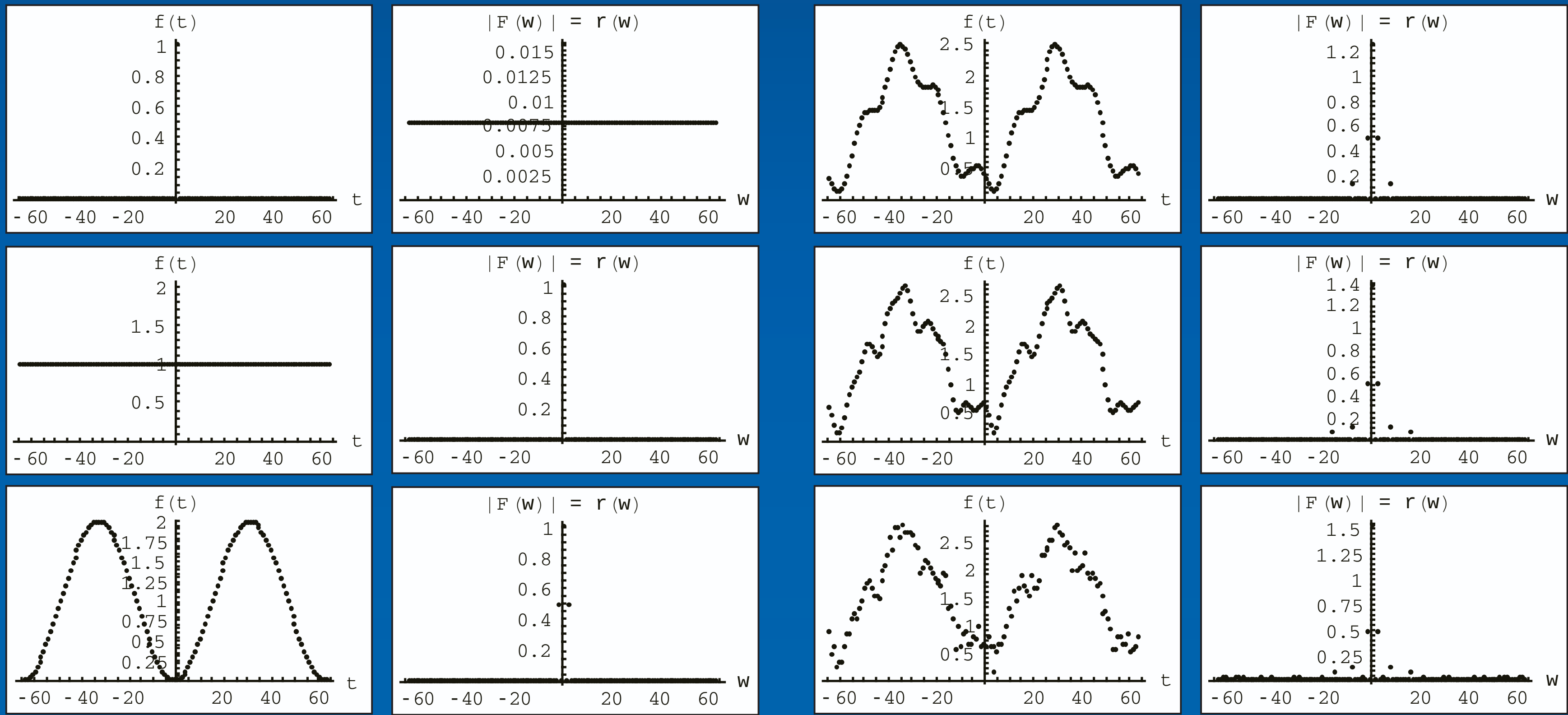
$$F(\omega)e^{2\pi i\omega\frac{t}{N}} + F(-\omega)e^{-2\pi i\omega\frac{t}{N}} = 2\rho(\omega)\cos 2\pi(\omega\frac{t}{N} - \phi(\omega))$$

$$F(\omega) = \rho(\omega)e^{-2\pi i\phi(\omega)} \qquad F(-\omega) = \rho(\omega)e^{2\pi i\phi(\omega)}$$

TRASFORMATA DI FOURIER

(una variabile discreta)

$$\omega = N/l$$



FFT = Fast Fourier Transform (per $N = 2^k$)

TRASFORMATA DI FOURIER

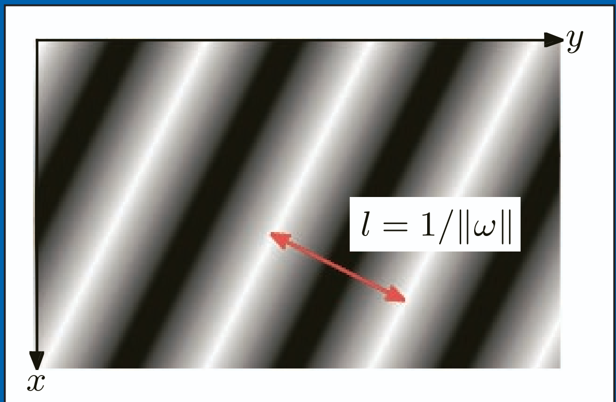
(due variabili continue)

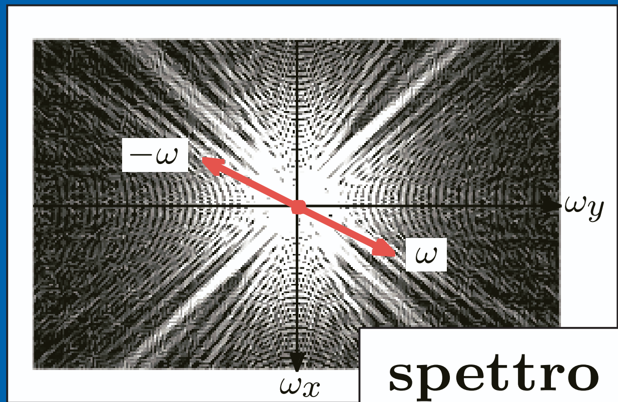
$$\boldsymbol{f(x,y) \;\rightsquigarrow\; F(\omega_x,\omega_y) = \iint_{-\infty}^{+\infty} f(x,y)e^{-2\pi i(\omega_x x+\omega_y y)}dxdy}$$

$$f(x,y) = \iint_{-\infty}^{+\infty} F(\omega)e^{2\pi i(\omega_x x+\omega_y y)}d\omega_x d\omega_y$$

$$= 2\int_{-\infty}^{+\infty}\int_{0}^{\infty}\rho(\omega)\cos 2\pi(\omega_x x+\omega_y y-\phi(\omega))d\omega_x d\omega_y$$

$$F(\pm\omega) = \rho(\omega)e^{\mp 2\pi i\phi(\omega)}$$





spettro di potenza

TRASFORMATA DI FOURIER
(due variabili discrete)

$$\boldsymbol{f(x,y) \rightsquigarrow F(\omega_x,\omega_y) = \frac{1}{N_x N_y}\sum_{x=1}^{N_x}\sum_{y=1}^{N_y} f(x,y)\, e^{-2\pi i\left(\omega_x \frac{x}{N_x}+\omega_y \frac{y}{N_y}\right)}}$$

$$f(x,y) = \sum_{\omega_x=[-N_x/2]}^{[N_x/2]}\ \sum_{\omega_y=[-N_y/2]}^{[N_y/2]} F(\omega_x,\omega_y)\, e^{2\pi i\left(\omega_x \frac{x}{N_x}+\omega_y \frac{y}{N_y}\right)}$$

$$F(0,0) = (f(1,1)+f(1,2)+\ldots+f(N_x,N_y))/N_x N_y$$

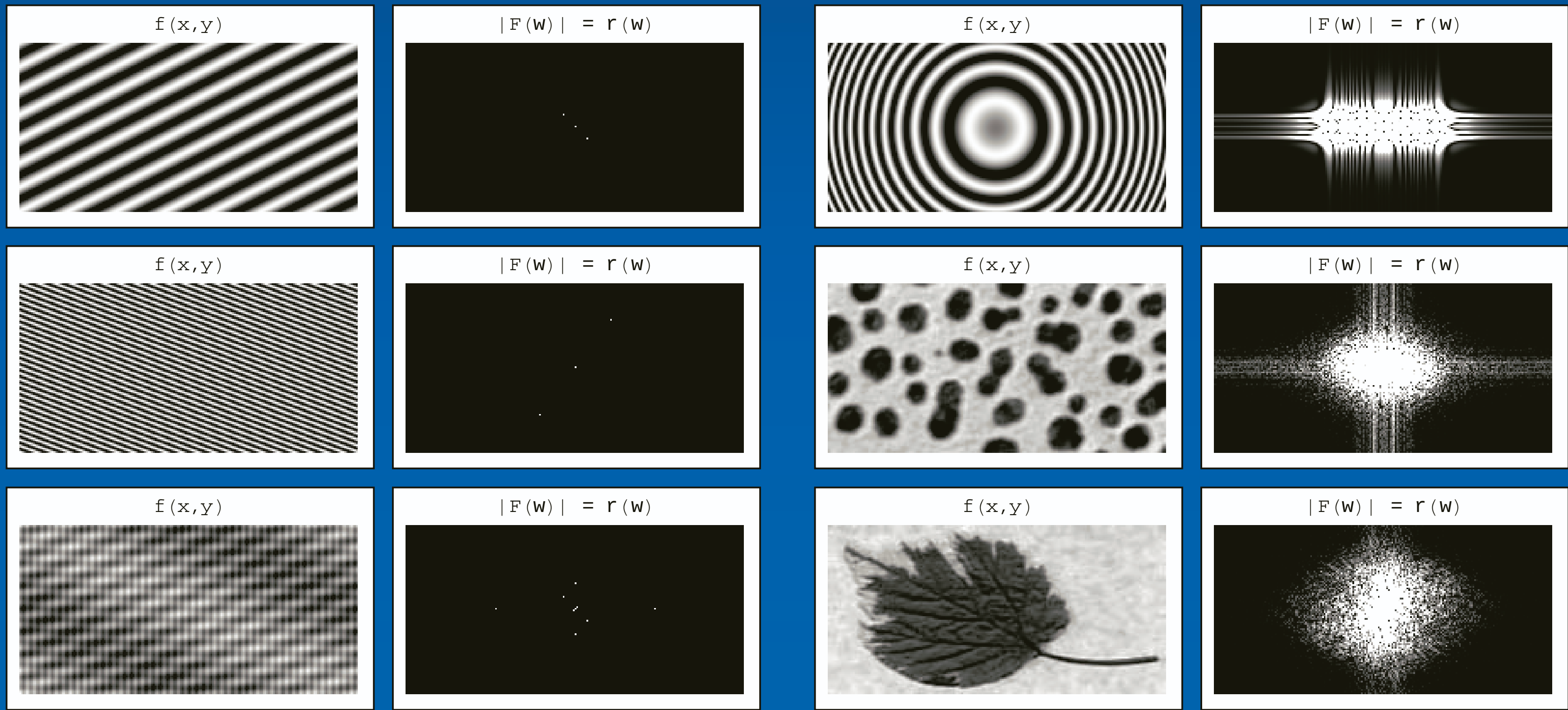
$$F(\omega)e^{2\pi i\left(\omega_x \frac{x}{N_x}+\omega_y \frac{y}{N_y}\right)} + F(-\omega)e^{-2\pi i\left(\omega_x \frac{x}{N_x}+\omega_y \frac{y}{N_y}\right)} =$$

$$= \rho(\omega)\cos 2\pi\left(\omega_x \frac{x}{N_x}+\omega_y \frac{y}{N_y} - \phi(\omega)\right)$$

$$F(\pm\omega) = \rho(\omega)e^{\mp 2\pi i \phi(\omega)}$$

TRASFORMATA DI FOURIER
(due variabili discrete)

$$\omega_x = N_x / l_x \qquad \omega_y = N_y / l_y$$

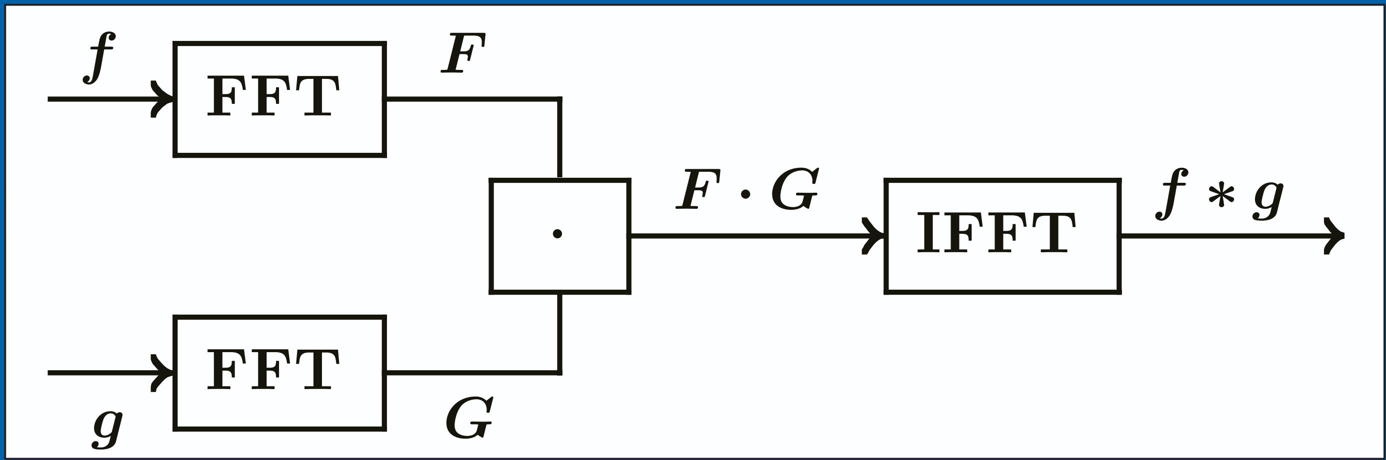


FFT = Fast Fourier Transform (per $N_x, N_y = 2^k$)

TEOREMA DI CONVOLUZIONE

$$(h(x,y))_{x,y} = (f(x,y))_{x,y} * (g(x,y))_{x,y}$$
$$\Updownarrow$$
$$(H(\omega_x,\omega_y))_{\omega_x,\omega_y} = (F(\omega_x,\omega_y) \cdot G(\omega_x,\omega_y))_{\omega_x,\omega_y}$$

$$(h(x,y))_{x,y} = (f(x,y) \cdot g(x,y))_{x,y}$$
$$\Updownarrow$$
$$(H(\omega_x,\omega_y))_{\omega_x,\omega_y} = (F(\omega_x,\omega_y))_{\omega_x,\omega_y} * (G(\omega_x,\omega_y))_{\omega_x,\omega_y}$$



FILTRI “PASSA BASSO”

Filtri di media

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

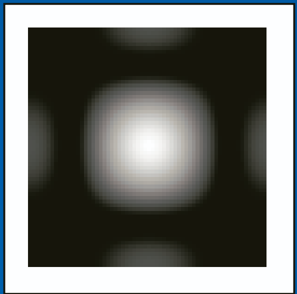
$$\frac{1}{25}\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

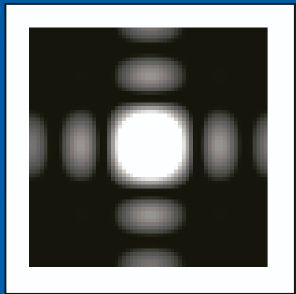
Filtri gaussiani

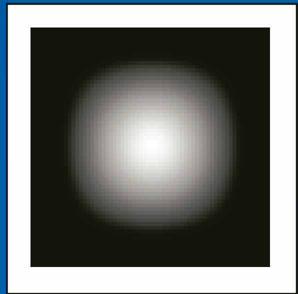
$$\frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

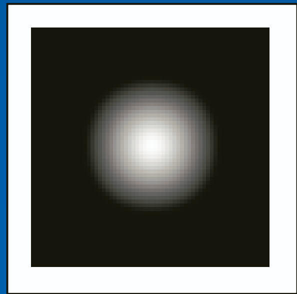
$$\frac{1}{256}\begin{bmatrix} 1 & 4 & 6 & 4 & 1 \\ 4 & 16 & 24 & 16 & 4 \\ 6 & 24 & 36 & 24 & 6 \\ 4 & 16 & 24 & 16 & 4 \\ 1 & 4 & 6 & 4 & 1 \end{bmatrix}$$

Nuclei di convoluzione
$(n(x,y))_{x,y}$







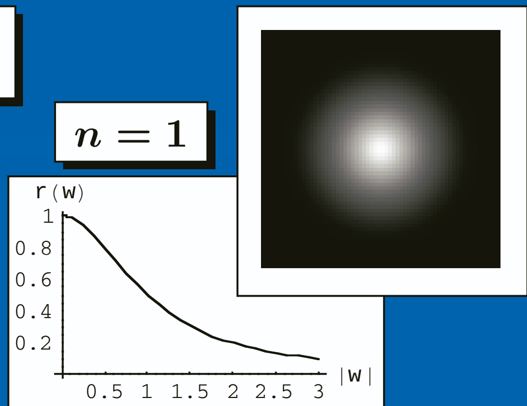


Fattori di potenza
$(F(\omega_x,\omega_y))_{\omega_x,\omega_y}$
$= \mathrm{FT}((n(x,y))_{x,y})$

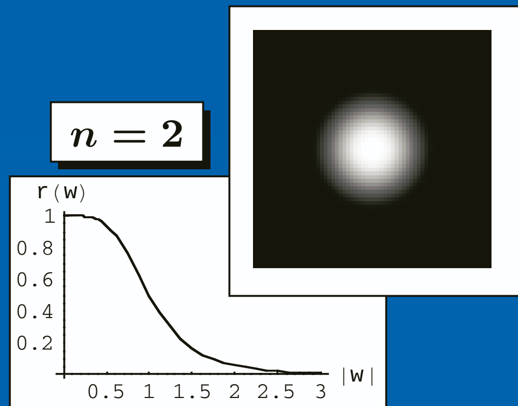
Filtri di Butterworth

$$F(\omega) = \frac{1}{1+|\omega|^{2n}}$$

$n = 1$



$n = 2$



FILTRI "PASSA ALTO"

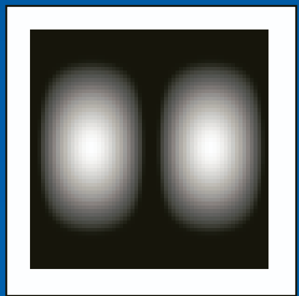
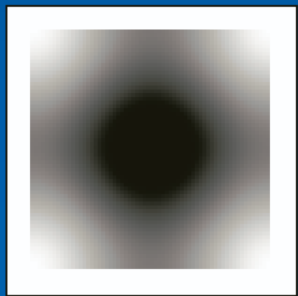
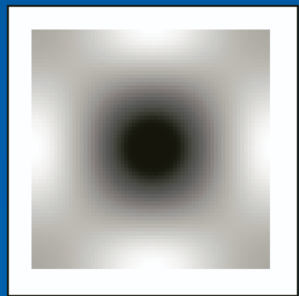
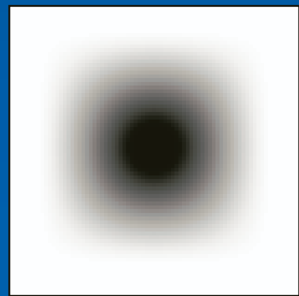
Filtri ∇_x, ∇_y e Δ

$$\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

Filtri sharpening

$$\begin{bmatrix} -1 & -1 & -1 \\ -1 & 9 & -1 \\ -1 & -1 & -1 \end{bmatrix} \quad \begin{bmatrix} -1 & -2 & -1 \\ -2 & 13 & -2 \\ -1 & -2 & -1 \end{bmatrix}$$

Nuclei di convoluzione
$(n(x, y))_{x,y}$

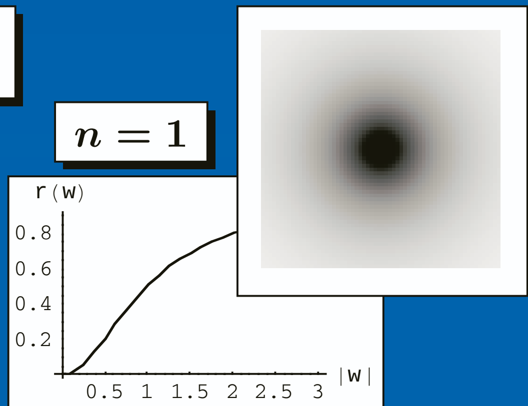





Fattori di potenza
$(F(\omega_x, \omega_y))_{\omega_x,\omega_y}$
$= \mathrm{FT}((n(x, y))_{x,y})$

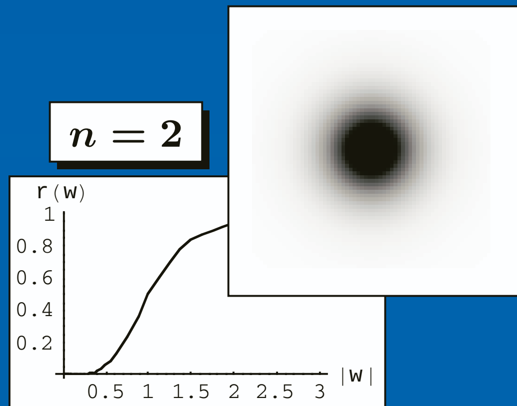
Filtri di Butterworth

$$F(\omega) = \frac{1}{1 + (r/|\omega|)^{2n}}$$

$n = 1$

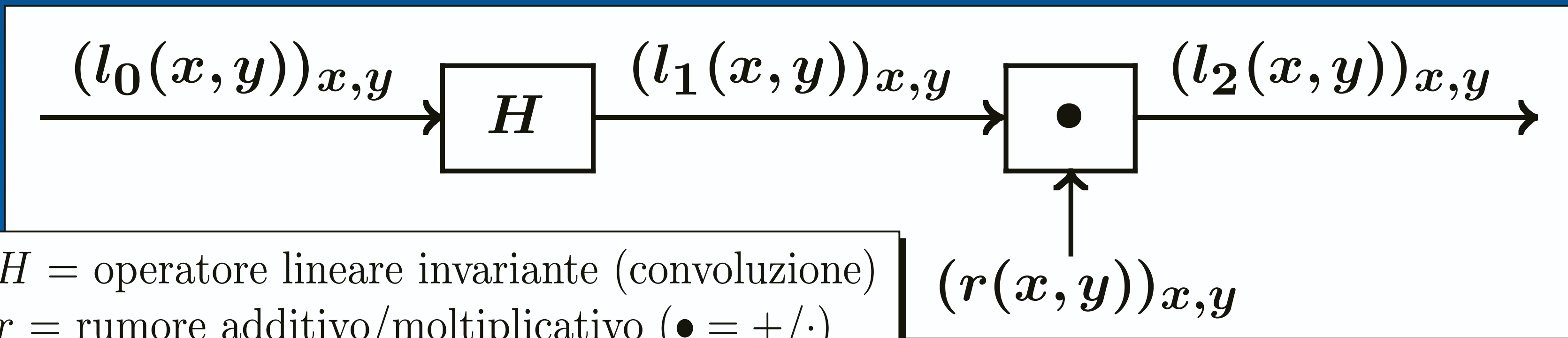


$n = 2$



RIPRISTINO DI IMMAGINI

Modello di deterioramento



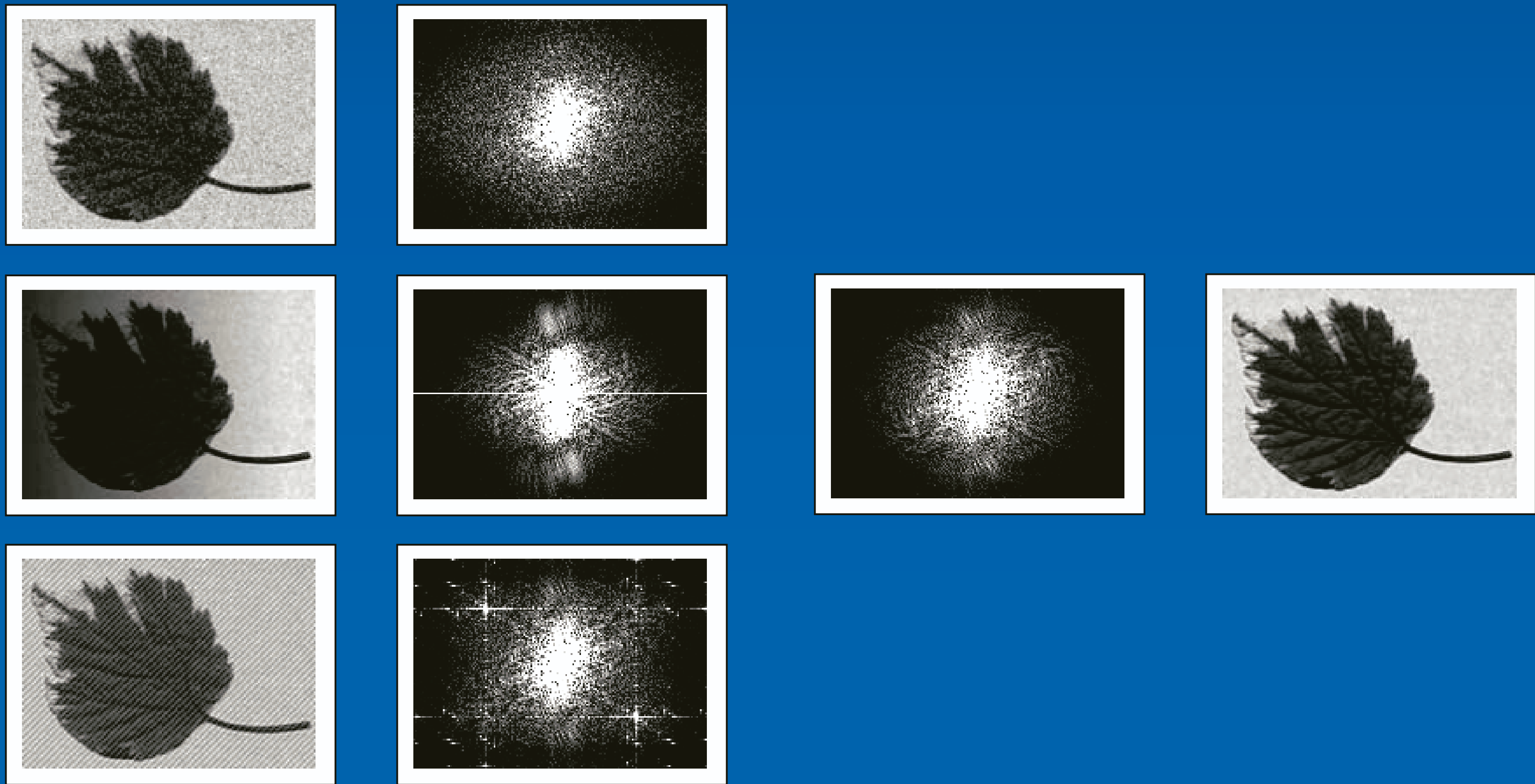
H = operatore lineare invariante (convoluzione)
r = rumore additivo/moltiplicativo ($\bullet = +/\cdot$)

Processo di ripristino

Filtraggio del rumore $\quad (l_2(x,y))_{x,y} \rightsquigarrow (\widehat{l}_1(x,y))_{x,y}$

Deconvoluzione $\quad (\widehat{l}_1(x,y))_{x,y} \rightsquigarrow (\widehat{l}_0(x,y))_{x,y}$

FILTRAGGIO DEL RUMORE



DECONV OLUZIONE

H

$$(l_1(x,y))_{x,y} = (l_0(x,y))_{x,y} * (n(x,y))_{x,y}$$
$$(L_1(\omega x,\omega y))_{\omega x,\omega y} = (L_0(\omega x,\omega y)\cdot F(\omega x,\omega y))_{\omega x,\omega y}$$

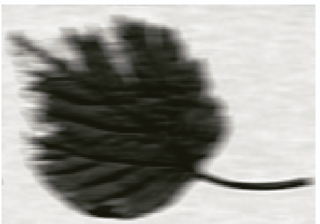
F

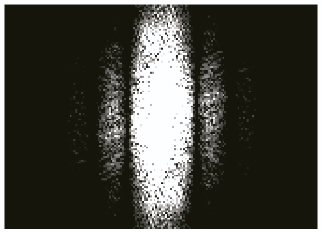
H^{-1}

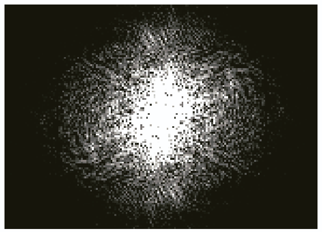
$$(l_0(x,y))_{x,y} = (l_1(x,y))_{x,y} * (n'(x,y))_{x,y}$$
$$(L_0(\omega x,\omega y))_{\omega x,\omega y} = (L_1(\omega x,\omega y)\,/\,F(\omega x,\omega y))_{\omega x,\omega y}$$

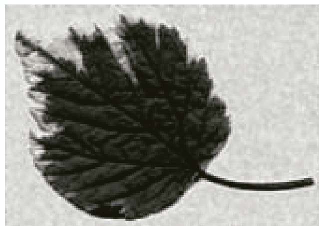
$1/F$

$(\widehat{l}_1(x,y))_{x,y}$ → **FFT** → **$1/F(\omega x,\omega y)_{\omega x,\omega y}$** → **IFFT** → $(\widehat{l}_0(x,y))_{x,y}$



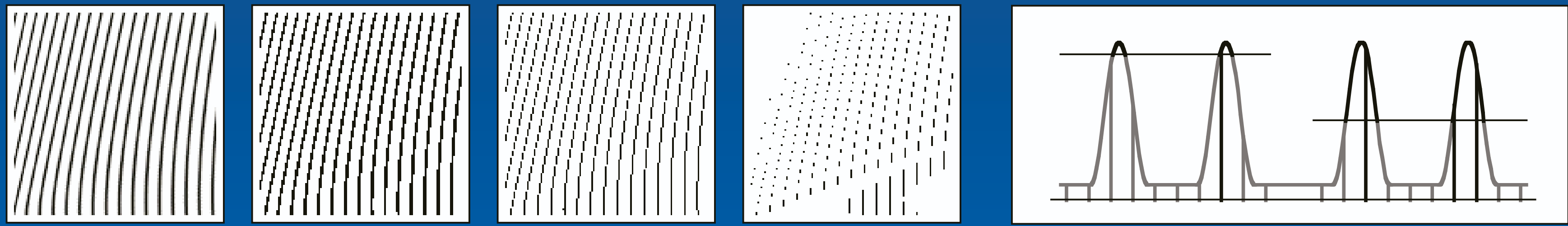




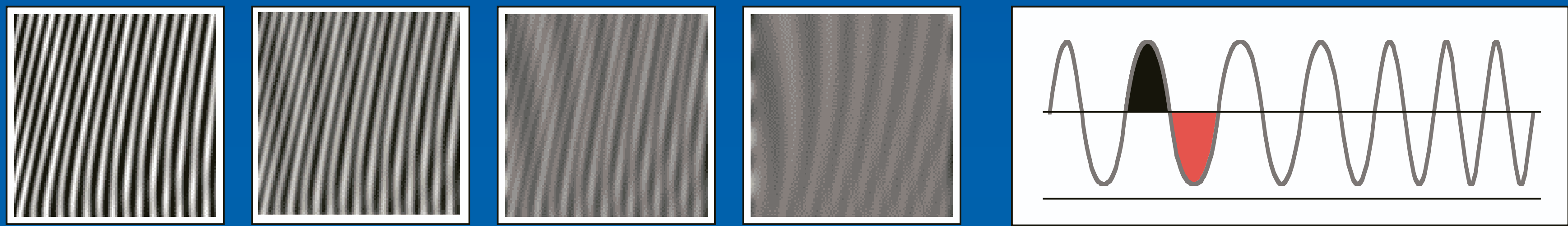


ALIASING

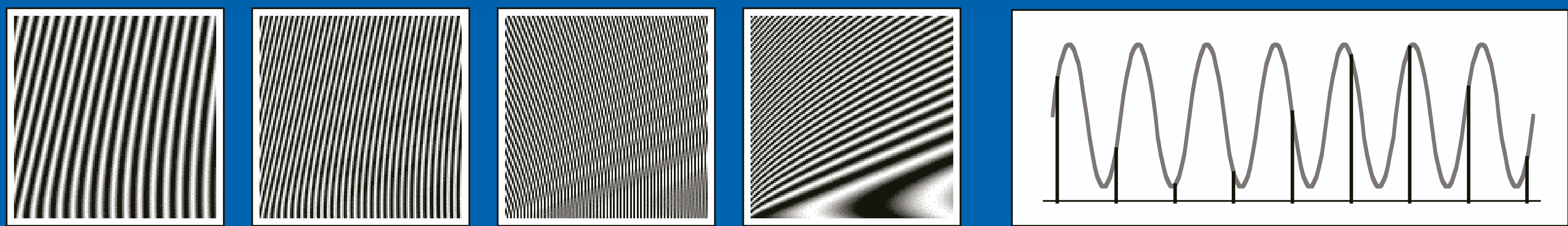
tresholding



filtraggio



campionamento



CAMPIONAMENTO

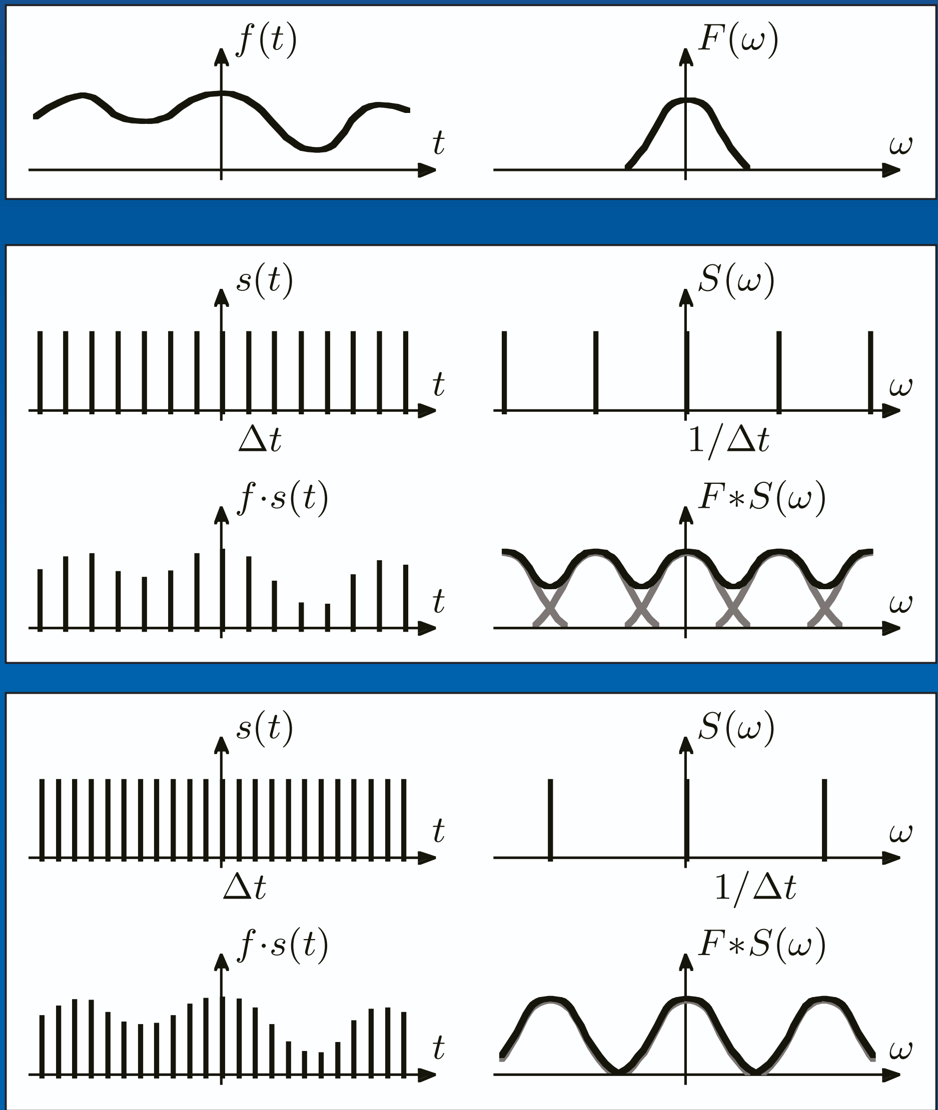


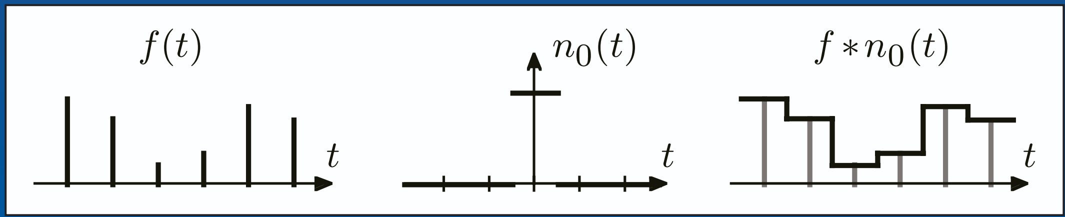
Immagine continua
(filtrata "passa basso")

Funzione di campionamento

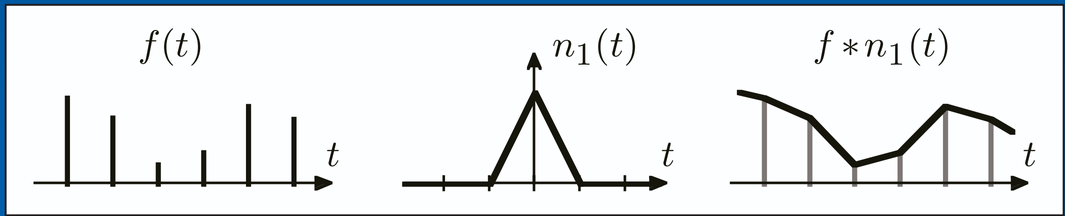
Immagine discreta

$$\boldsymbol{\Delta x < \min(1/2\omega_x)}$$
$$\boldsymbol{\Delta y < \min(1/2\omega_y)}$$

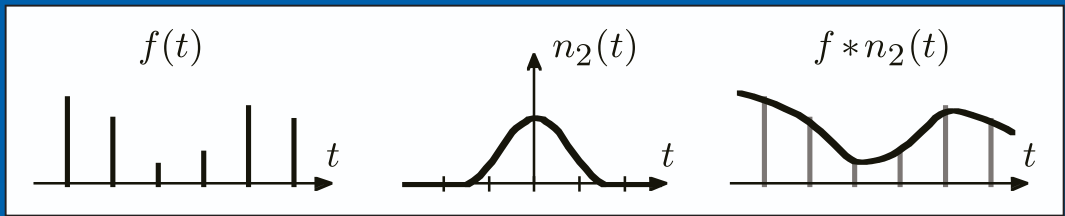
RICOSTRUZIONE



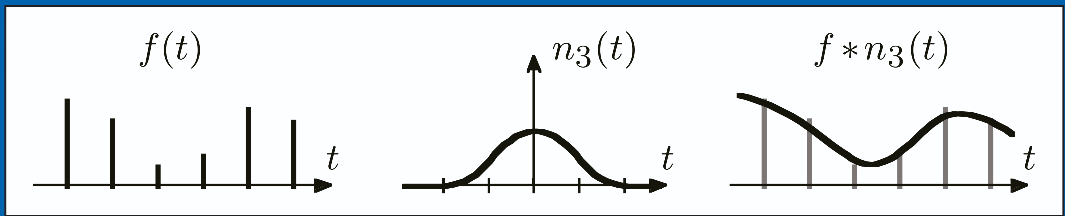
Punto più vicino



Interpolazione (bi-)lineare

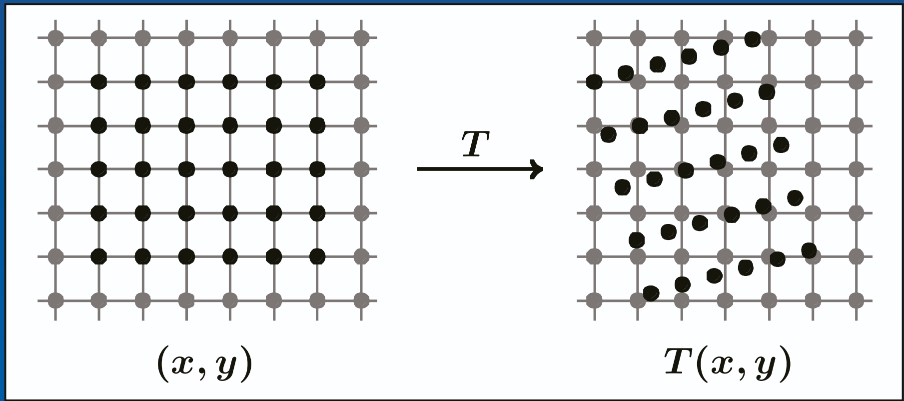


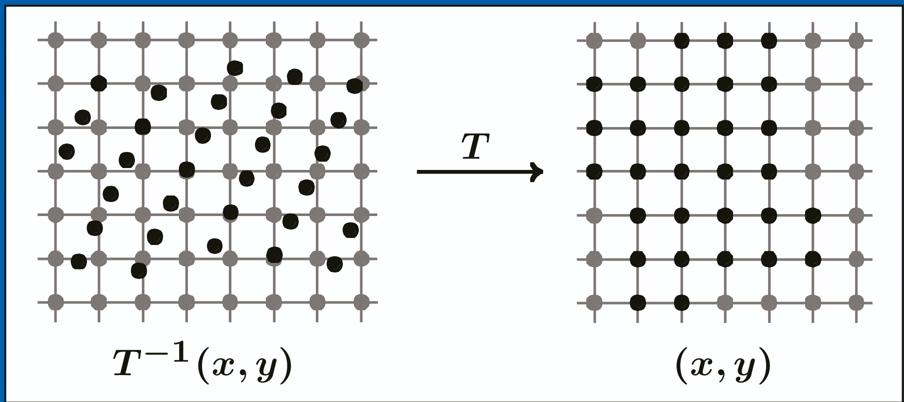
Interpolazione (bi-)quadratica

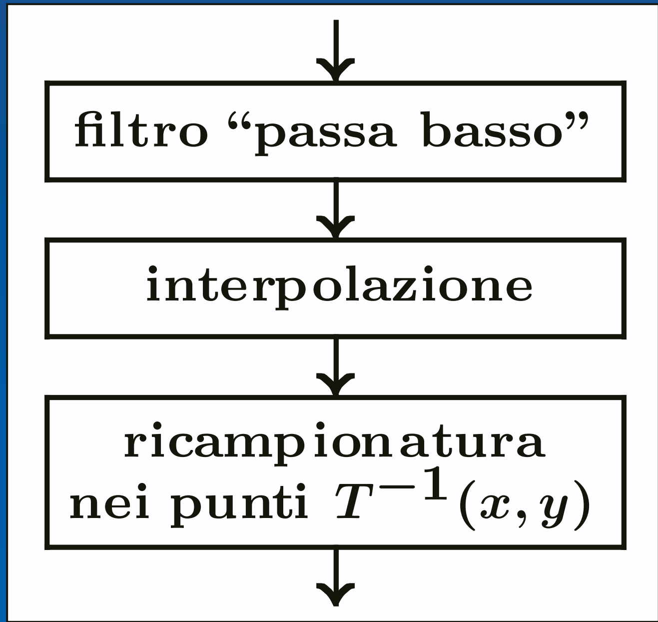


Interpolazione (bi-)cubica

TRASFORMAZIONI







$$T = \begin{cases} \text{isometria (traslazione, rotazione, simmetria)} \\ \text{scalatura (uniforme, non uniforme)} \\ \text{distorsione (lineare, non lineare)} \end{cases}$$

Multirisoluzione