Pentagono

In[1]:=
```
addsin = Sin[x + y] -> Sin[x] Cos[y] + Sin[y] Cos[x]
```

Out[1]=
$$\text{Sin}[x + y] \to \text{Cos}[y]\ \text{Sin}[x] + \text{Cos}[x]\ \text{Sin}[y]$$

In[2]:=
```
addcos = Cos[x + y] -> Cos[x] Cos[y] - Sin[y] Sin[x]
```

Out[2]=
$$\text{Cos}[x + y] \to \text{Cos}[x]\ \text{Cos}[y] - \text{Sin}[x]\ \text{Sin}[y]$$

In[3]:=
```
sin2 = addsin /. y -> x
```

Out[3]=
$$\text{Sin}[2\ x] \to 2\ \text{Cos}[x]\ \text{Sin}[x]$$

In[4]:=
```
cos2 = addcos /. y -> x
```

Out[4]=
$$\text{Cos}[2\ x] \to \text{Cos}[x]^2 - \text{Sin}[x]^2$$

In[5]:=
```
sin3 = addsin /. y -> 2x
```

Out[5]=
$$\text{Sin}[3\ x] \to \text{Cos}[2\ x]\ \text{Sin}[x] + \text{Cos}[x]\ \text{Sin}[2\ x]$$

In[6]:=
```
sin3 = sin3 /. {sin2,cos2}
```

Out[6]=
$$\text{Sin}[3\ x] \to 2\ \text{Cos}[x]^2\ \text{Sin}[x] + \text{Sin}[x]\ (\text{Cos}[x]^2 - \text{Sin}[x]^2)$$

In[7]:=
```
sin3 = ExpandAll[sin3]
```

Out[7]=
$$\text{Sin}[3\ x] \to 3\ \text{Cos}[x]^2\ \text{Sin}[x] - \text{Sin}[x]^3$$

In[8]:=
```
cos3 = addcos /. y -> 2x
```

Out[8]=
$$\text{Cos}[3\ x] \to \text{Cos}[x]\ \text{Cos}[2\ x] - \text{Sin}[x]\ \text{Sin}[2\ x]$$

In[9]:=
```
cos3 = cos3 /. {sin2,cos2}
```

Out[9]=
$$\text{Cos}[3\ x] \to -2\ \text{Cos}[x]\ \text{Sin}[x]^2 + \text{Cos}[x]\ (\text{Cos}[x]^2 - \text{Sin}[x]^2)$$

In[10]:=

```
cos3 = ExpandAll[cos3]
```

Out[10]=

$$\text{Cos}[3\,x] \to \text{Cos}[x]^3 - 3\,\text{Cos}[x]\,\text{Sin}[x]^2$$

In[11]:=

```
sin4 = addsin /. y -> 3x
```

Out[11]=

$$\text{Sin}[4\,x] \to \text{Cos}[3\,x]\,\text{Sin}[x] + \text{Cos}[x]\,\text{Sin}[3\,x]$$

In[12]:=

```
sin4 = sin4 /. {sin3,cos3}
```

Out[12]=

$$\text{Sin}[4\,x] \to \text{Sin}[x]\,(\text{Cos}[x]^3 - 3\,\text{Cos}[x]\,\text{Sin}[x]^2) + \text{Cos}[x]\,(3\,\text{Cos}[x]^2\,\text{Sin}[x] - \text{Sin}[x]^3)$$

In[13]:=

```
sin4 = ExpandAll[sin4]
```

Out[13]=

$$\text{Sin}[4\,x] \to 4\,\text{Cos}[x]^3\,\text{Sin}[x] - 4\,\text{Cos}[x]\,\text{Sin}[x]^3$$

In[14]:=

```
cos4 = addcos /. y -> 3x
```

Out[14]=

$$\text{Cos}[4\,x] \to \text{Cos}[x]\,\text{Cos}[3\,x] - \text{Sin}[x]\,\text{Sin}[3\,x]$$

In[15]:=

```
cos4 = cos4 /. {sin3,cos3}
```

Out[15]=

$$\text{Cos}[4\,x] \to \text{Cos}[x]\,(\text{Cos}[x]^3 - 3\,\text{Cos}[x]\,\text{Sin}[x]^2) - \text{Sin}[x]\,(3\,\text{Cos}[x]^2\,\text{Sin}[x] - \text{Sin}[x]^3)$$

In[16]:=

```
cos4 = ExpandAll[cos4]
```

Out[16]=

$$\text{Cos}[4\,x] \to \text{Cos}[x]^4 - 6\,\text{Cos}[x]^2\,\text{Sin}[x]^2 + \text{Sin}[x]^4$$

In[17]:=

```
sin5 = addsin /. y -> 4x
```

Out[17]=

$$\text{Sin}[5\,x] \to \text{Cos}[4\,x]\,\text{Sin}[x] + \text{Cos}[x]\,\text{Sin}[4\,x]$$

In[18]:=

```
sin5 = sin5 /. {sin4,cos4}
```

Out[18]=

$$\text{Sin[5 x]} \to \text{Cos[x]}\,(4\,\text{Cos[x]}^3\,\text{Sin[x]} - 4\,\text{Cos[x]}\,\text{Sin[x]}^3) + \text{Sin[x]}\,(\text{Cos[x]}^4 - 6\,\text{Cos[x]}^2\,\text{Sin[x]}^2 + \text{Sin[x]}^4)$$

In[19]:=

```
sin5 = ExpandAll[sin5]
```

Out[19]=

$$\text{Sin[5 x]} \to 5\,\text{Cos[x]}^4\,\text{Sin[x]} - 10\,\text{Cos[x]}^2\,\text{Sin[x]}^3 + \text{Sin[x]}^5$$

In[20]:=

```
sin5 = sin5 /. Cos[x] -> Sqrt[1 - Sin[x]^2]
```

Out[20]=

$$\text{Sin[5 x]} \to \text{Sin[x]}^5 - 10\,\text{Sin[x]}^3\,(1 - \text{Sin[x]}^2) + 5\,\text{Sin[x]}\,(1 - \text{Sin[x]}^2)^2$$

In[21]:=

```
sin5 = ExpandAll[sin5]
```

Out[21]=

$$\text{Sin[5 x]} \to 5\,\text{Sin[x]} - 20\,\text{Sin[x]}^3 + 16\,\text{Sin[x]}^5$$

In[22]:=

```
equaz = Sin[5 x] == 0 /. sin5
```

Out[22]=

$$5\,\text{Sin[x]} - 20\,\text{Sin[x]}^3 + 16\,\text{Sin[x]}^5 == 0$$

In[23]:=

```
equaz = equaz /. x -> Pi/5
```

Out[23]=

$$5\,\text{Sin}\left[\frac{\text{Pi}}{5}\right] - 20\,\text{Sin}\left[\frac{\text{Pi}}{5}\right]^3 + 16\,\text{Sin}\left[\frac{\text{Pi}}{5}\right]^5 == 0$$

In[24]:=

```
equaz = equaz /. Sin[Pi/5] -> s
```

Out[24]=

$$5\,s - 20\,s^3 + 16\,s^5 == 0$$

In[25]:=

```
soluzioni = Solve[equaz,s]
```

Out[25]=

$$\left\{\{s \to 0\}, \left\{s \to \frac{-\text{Sqrt}[5 - \text{Sqrt}[5]]}{2\,\text{Sqrt}[2]}\right\}, \left\{s \to \frac{\text{Sqrt}[5 - \text{Sqrt}[5]]}{2\,\text{Sqrt}[2]}\right\}, \left\{s \to \frac{-\text{Sqrt}[5 + \text{Sqrt}[5]]}{2\,\text{Sqrt}[2]}\right\}, \left\{s \to \frac{\text{Sqrt}[5 + \text{Sqrt}[5]]}{2\,\text{Sqrt}[2]}\right\}\right\}$$

In[26]:=

```
lato = 2 s /. soluzioni[[5]]
```

Out[26]=

$$\frac{\sqrt{5+\sqrt{5}}}{\sqrt{2}}$$