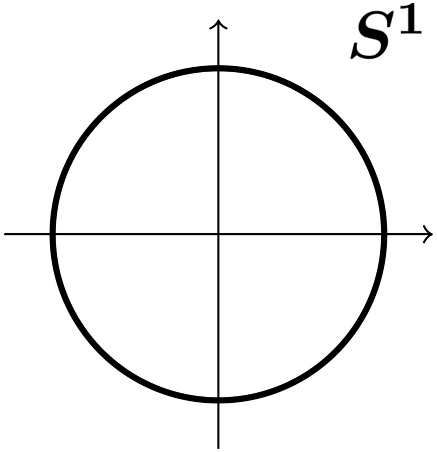
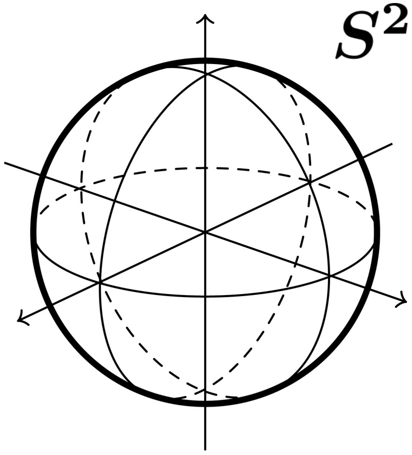
Da Poincaré a Perelman: un secolo di ordinaria matematica

Spazi euclidei e sfere



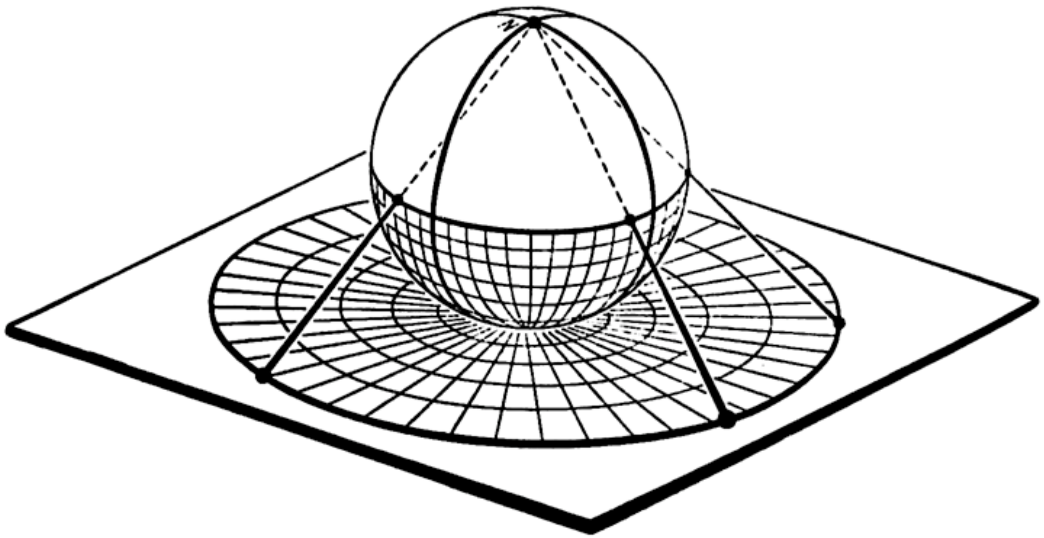
$S^1 \subset R^2$



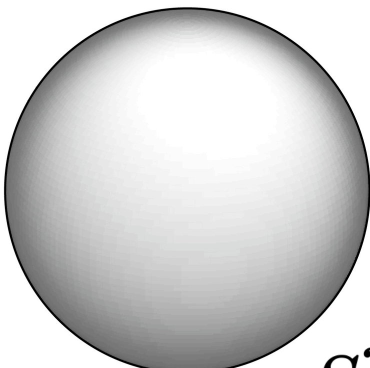
$S^2 \subset R^3$

$$S^n = \{x \in R^{n+1} \mid \|x\| = 1\}$$

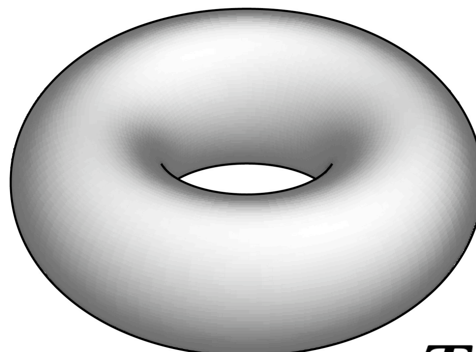
$$S^n \cong R^n \cup \{\infty\}$$



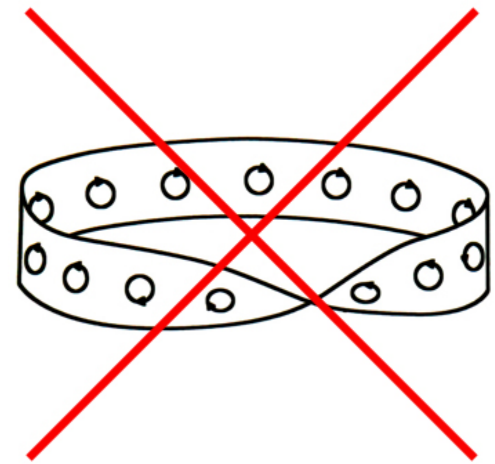
Varietà (chiuse orientabili)

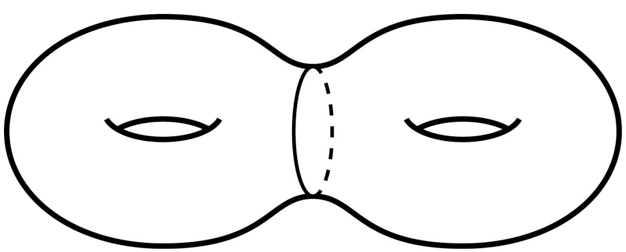


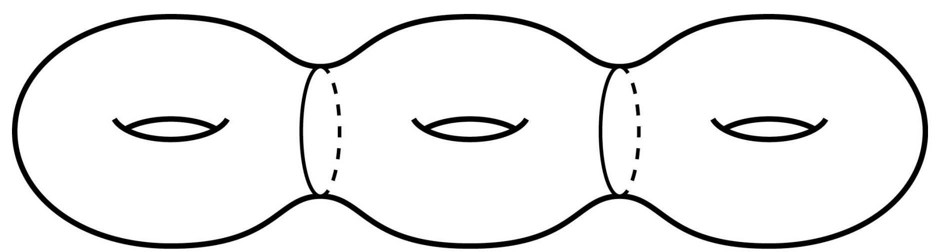
S^2



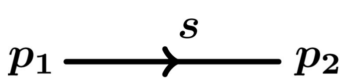
T^2



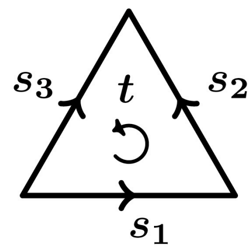




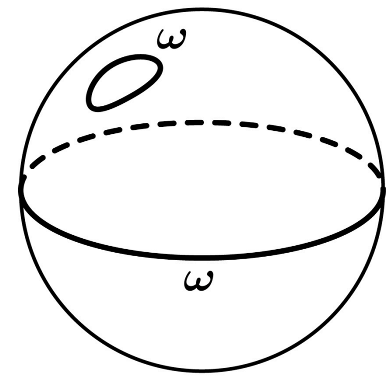
Triangolazioni e omologia



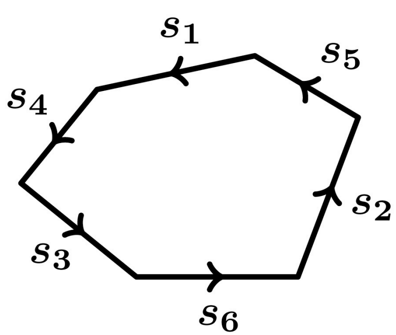
$\partial s = p_2 - p_1$



$\partial t = s_1 + s_2 - s_3$

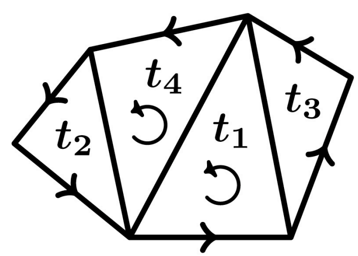


$\forall\, \omega \simeq * \; (\omega \sim 0)$



$\omega = s_1 + \ldots + s_6$

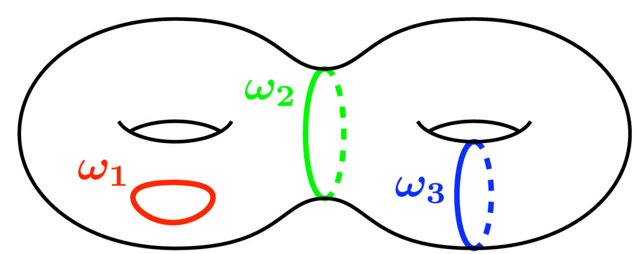
$\partial \omega = 0$



$c = t_1 + \ldots + t_4$

$\omega = \partial c$

$\omega_2 \sim 0 \; (\omega_2 \not\simeq *)$



$\omega_1 \simeq *$ $\qquad$ $\omega_3 \not\sim 0$

La congettura di Poincaré

Second Complément à l'Analysis Sit
at request of the President, J
June 30th, 1900.

Pour ne pas trop allonger ce travai
théorème suivant dont la démonstratio
loppements :—

***Tout polyèdre qui a tous ses nombre
tableaux T_q bilatères est simplement con
l'hypersphère.***

CINQUIÈME COMPLÉMENT À L'ANALYSIS SITUS.

Par M. H. Poincaré, à Paris.

Adunanza del 22 novembre 1903.

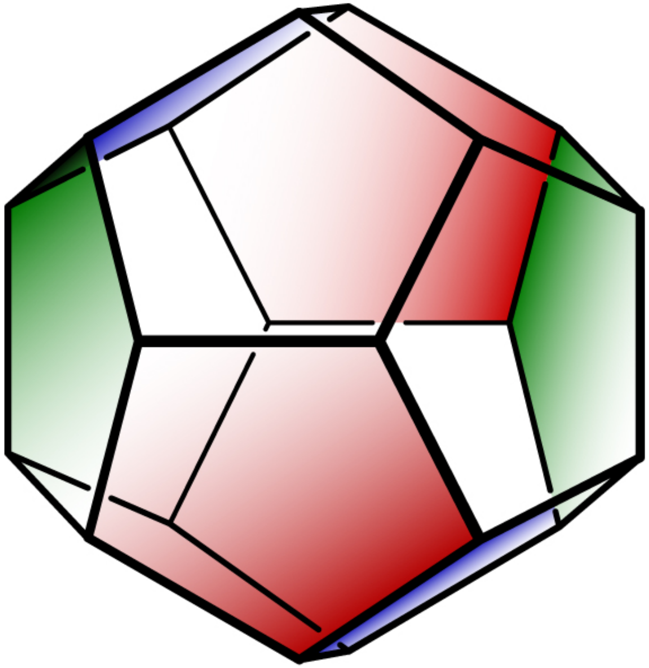
On pourrait alors se demander si la considération de ces coefficients suffit ; si une variété dont tous les nombres de Betti et les coefficients de torsion sont égaux à 1 est pour cela simplement connexe au sens propre du mot, c'est-à-dire homéomorphe à l'hypersphère ; ou si, au contraire, il est nécessaire, avant d'affirmer qu'une variété est simplement connexe, d'étudier son *groupe fondamental*, tel que je l'ai défini dans le « Journal de l'École Polytechnique », § 12, page 60.

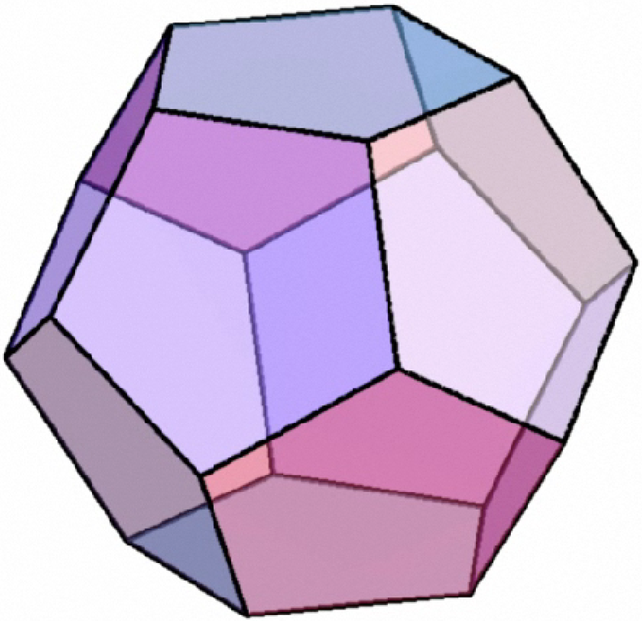
Nous pouvons maintenant répondre à cette question ; j'ai formé en effet un exemple d'une variété dont tous les nombres de Betti et les coefficients de torsion sont égaux à 1, et qui pourtant n'est pas simplement connexe.

Il resterait une question à traiter :

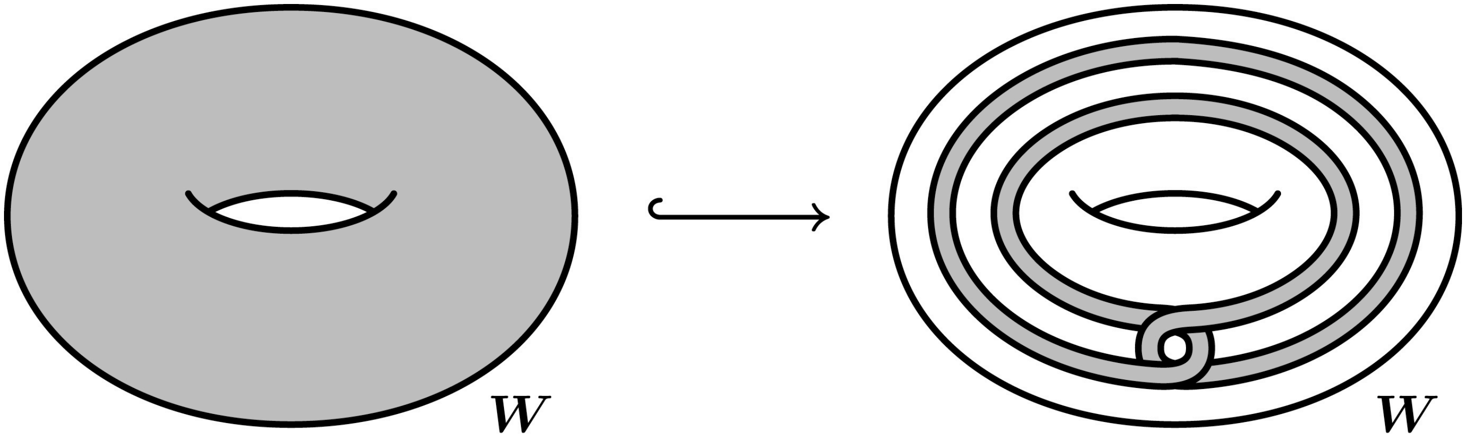
Est-il possible que le groupe fondamental de V se réduise à la substitution identique, et que pourtant V ne soit pas simplement connexe?

La sfera omologica





Lo spazio di Whitehead

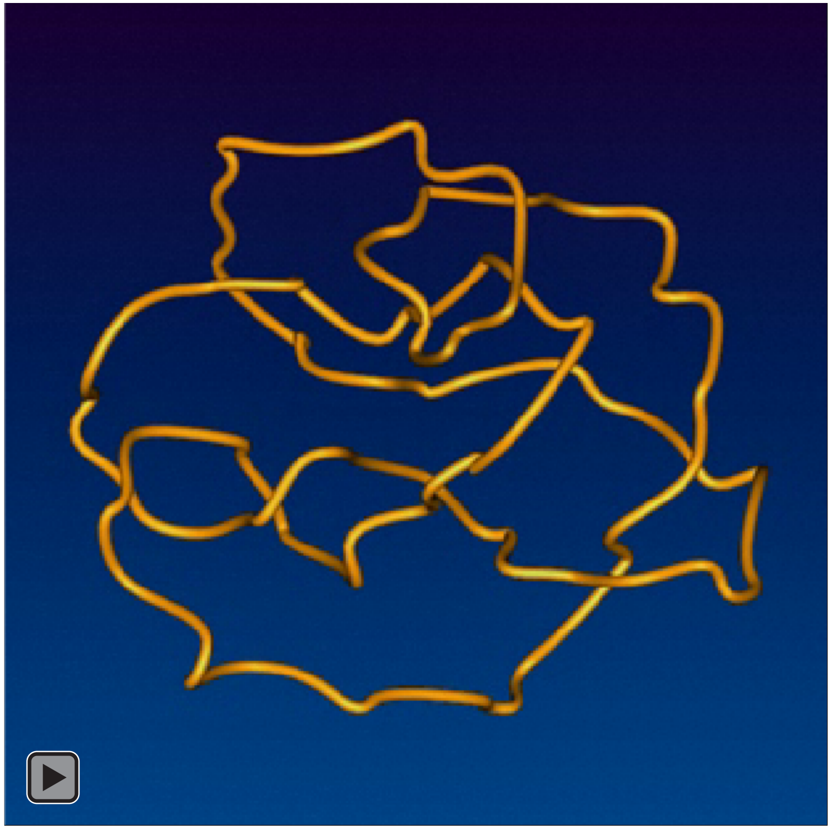


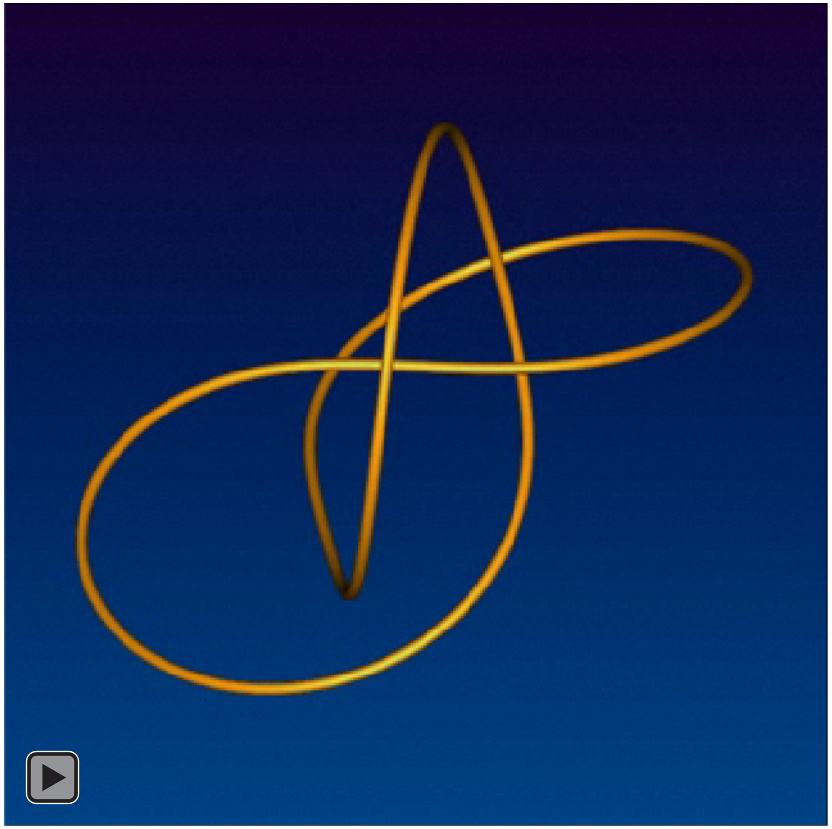
$$W_1 \hookrightarrow W_2 \hookrightarrow W_3 \hookrightarrow \cdots \hookrightarrow W_i \hookrightarrow W_{i+1} \hookrightarrow \cdots$$

$$W_\infty = W_1 \cup W_2 \cup \cdots \cup W_i \cup W_{i+1} \cup \cdots$$

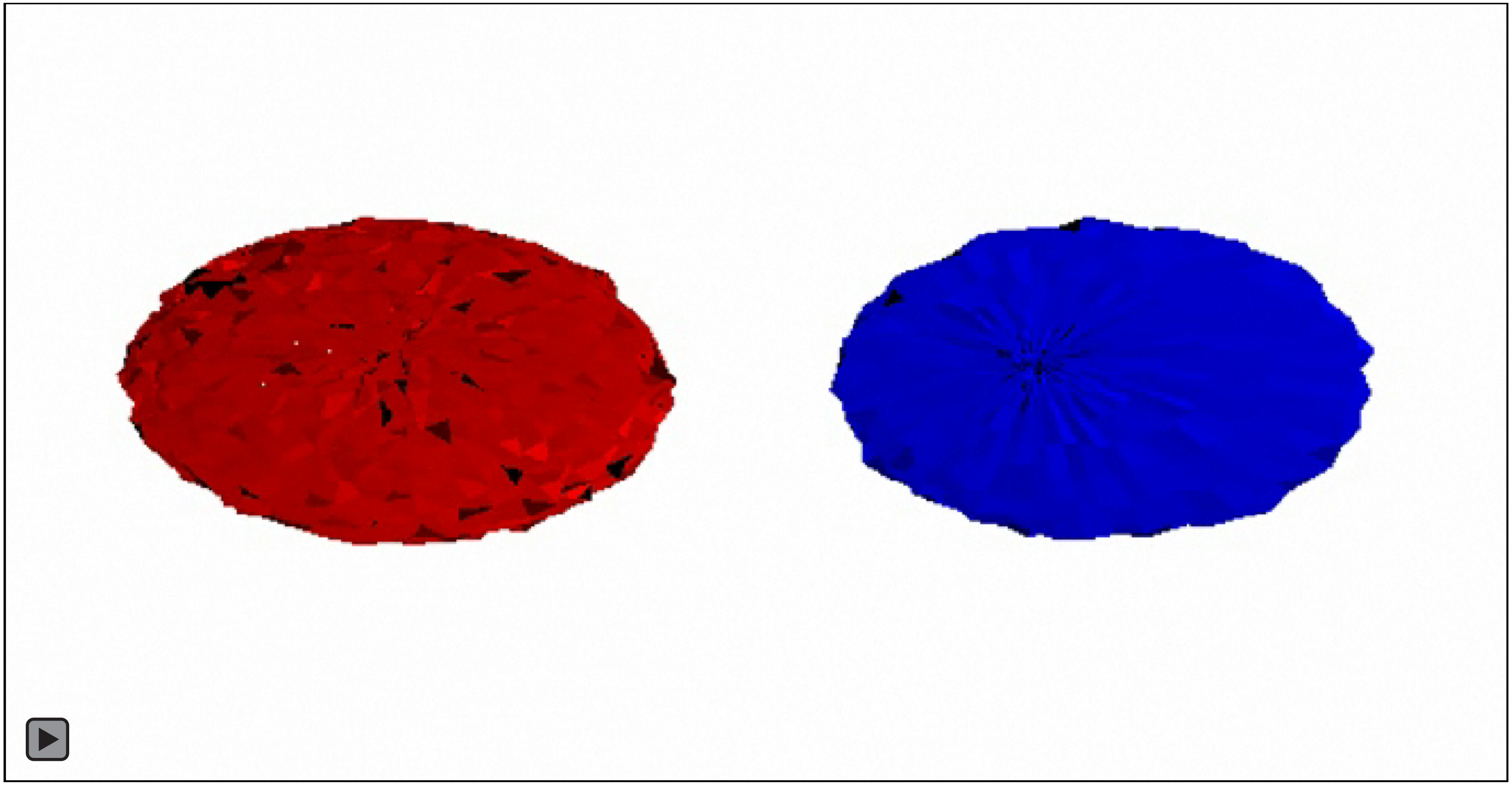
$$W_\infty \searrow * \quad (W_i \searrow * \text{ in } W_{i+1} \text{ per ogni } i)$$

Topologia e geometria

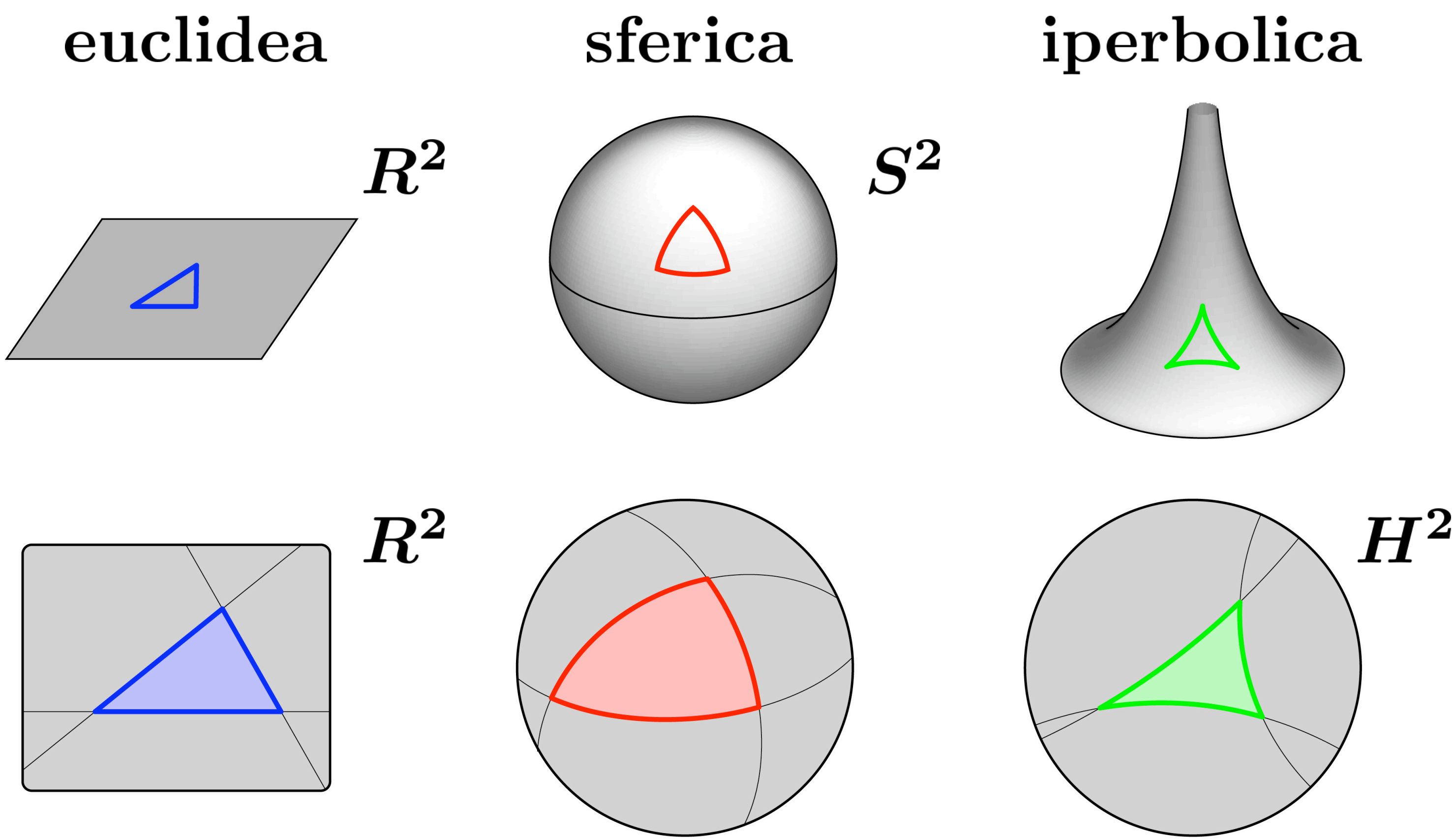




Topologia e geometria

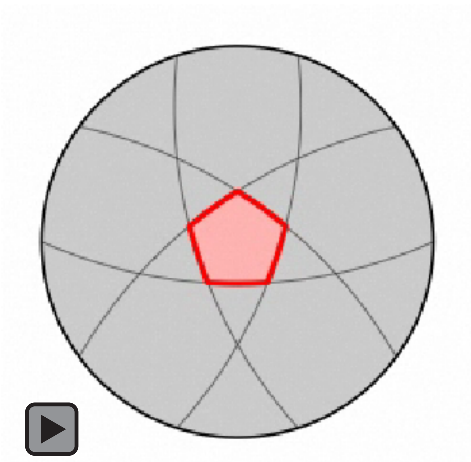


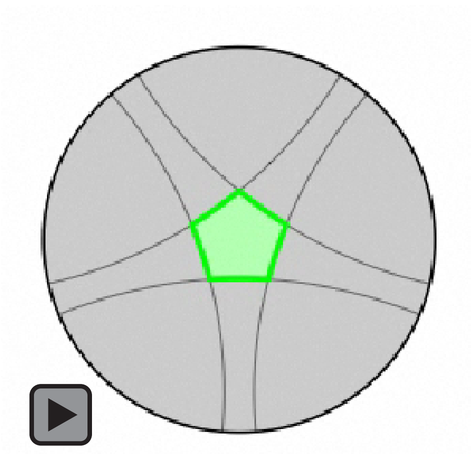
Le geometrie 2-dimensionali

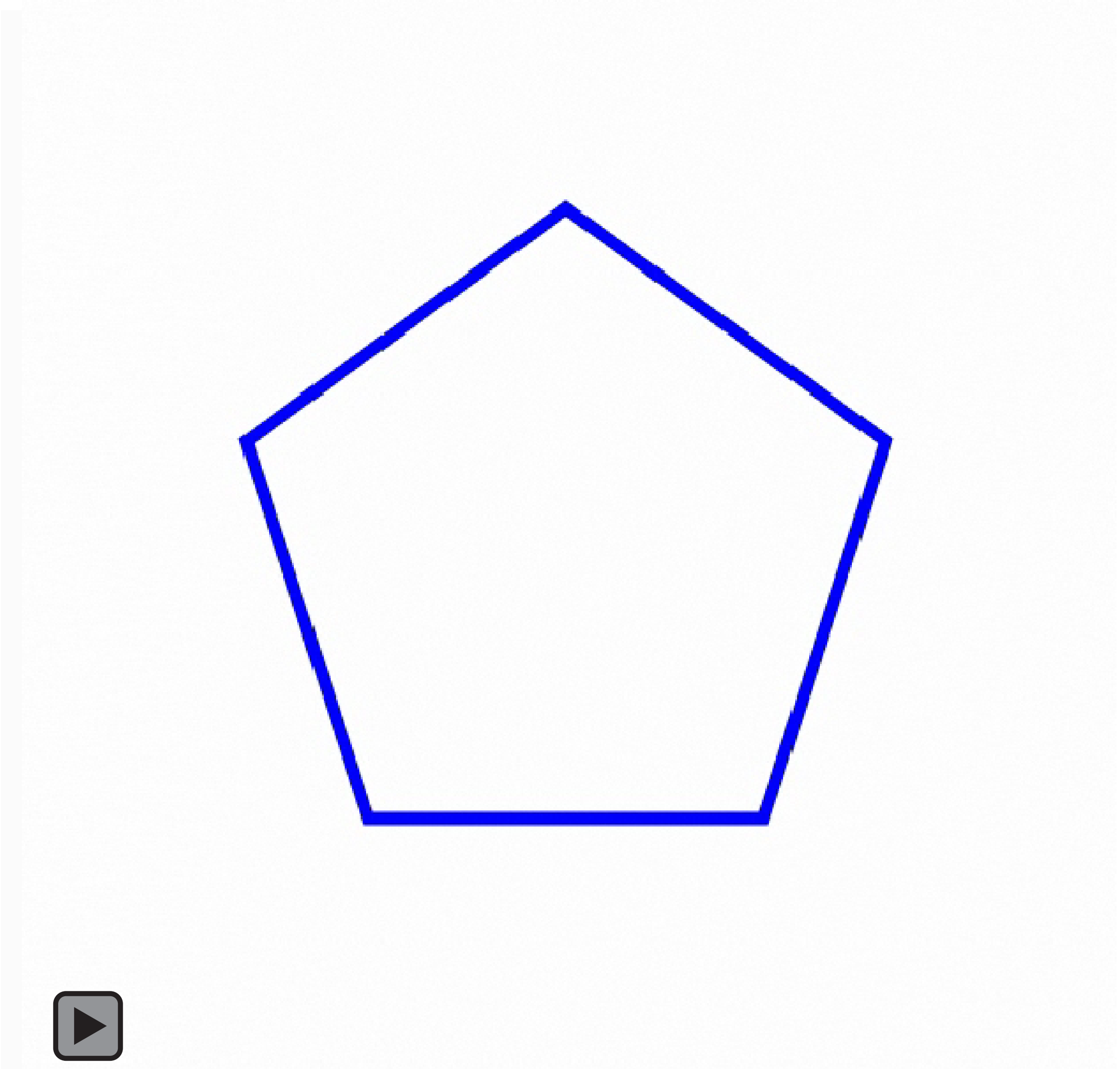


$$K = (\alpha + \beta + \gamma - \pi)/A$$

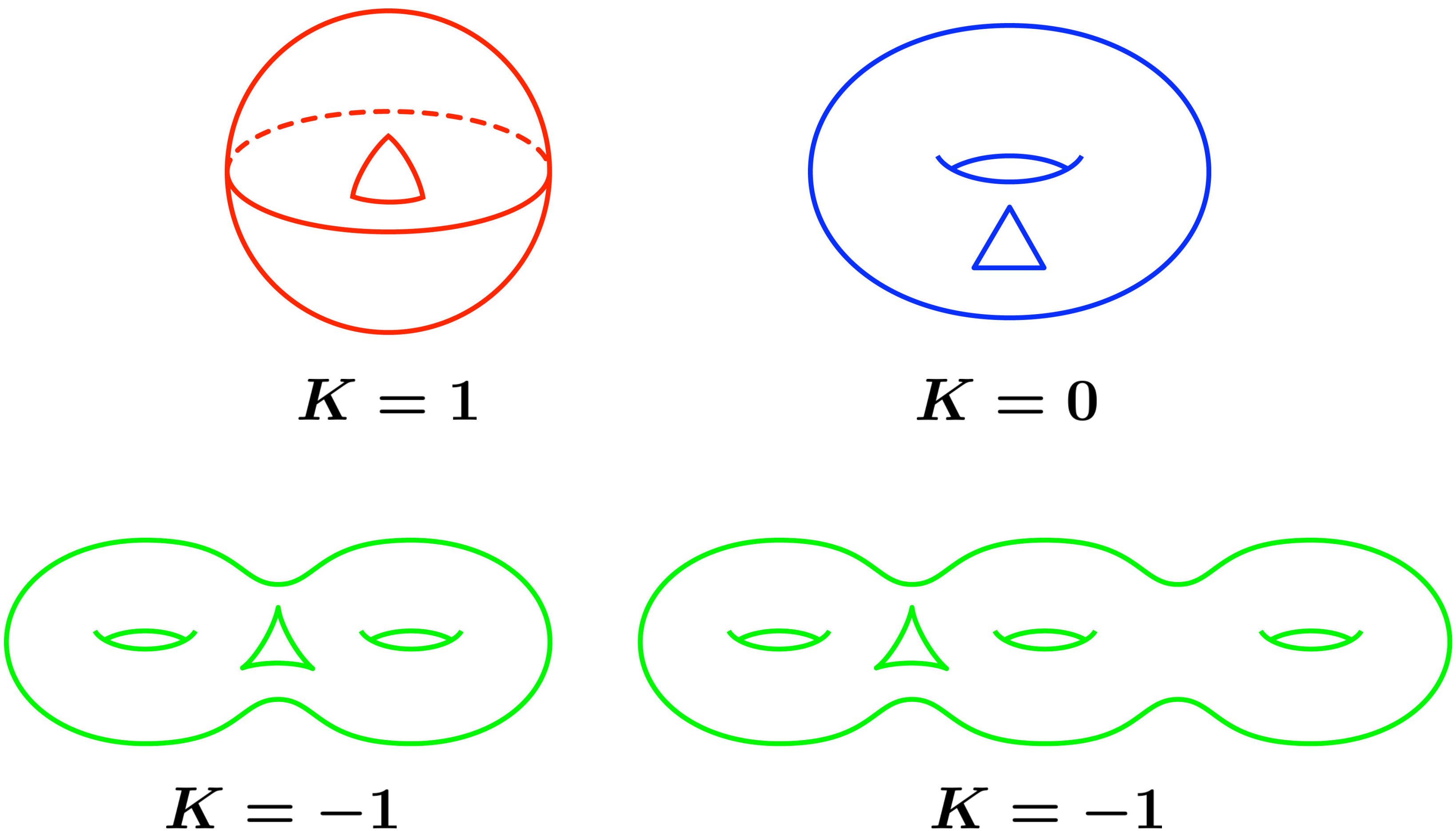
Le geometrie 2-dimensionali



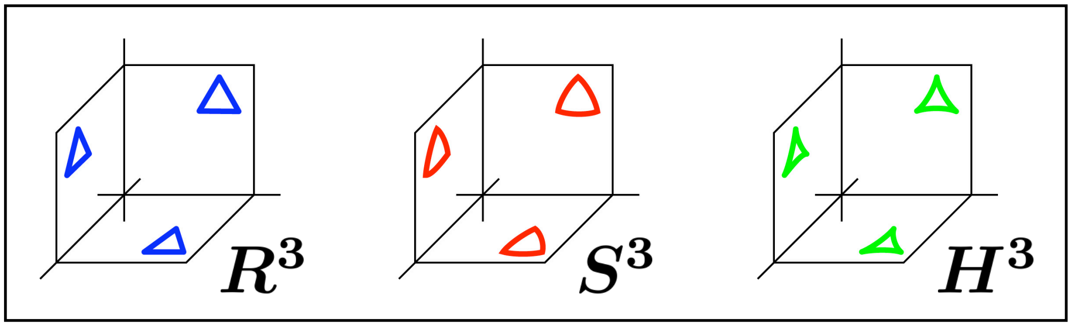




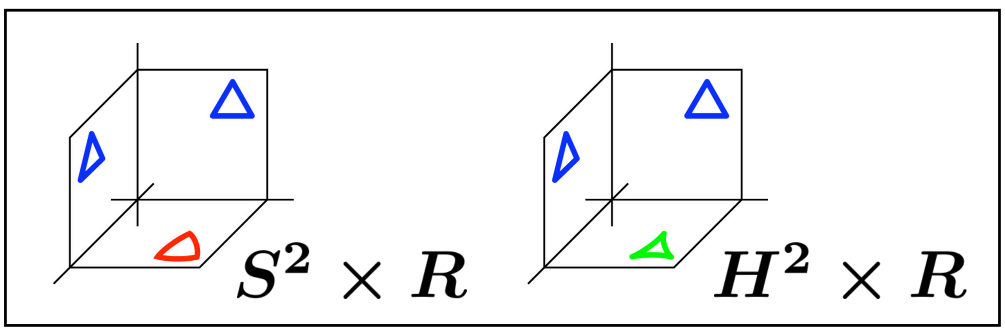
Il teorema di uniformizzazione



Le geometrie 3-dimensionali



spazi isotropi

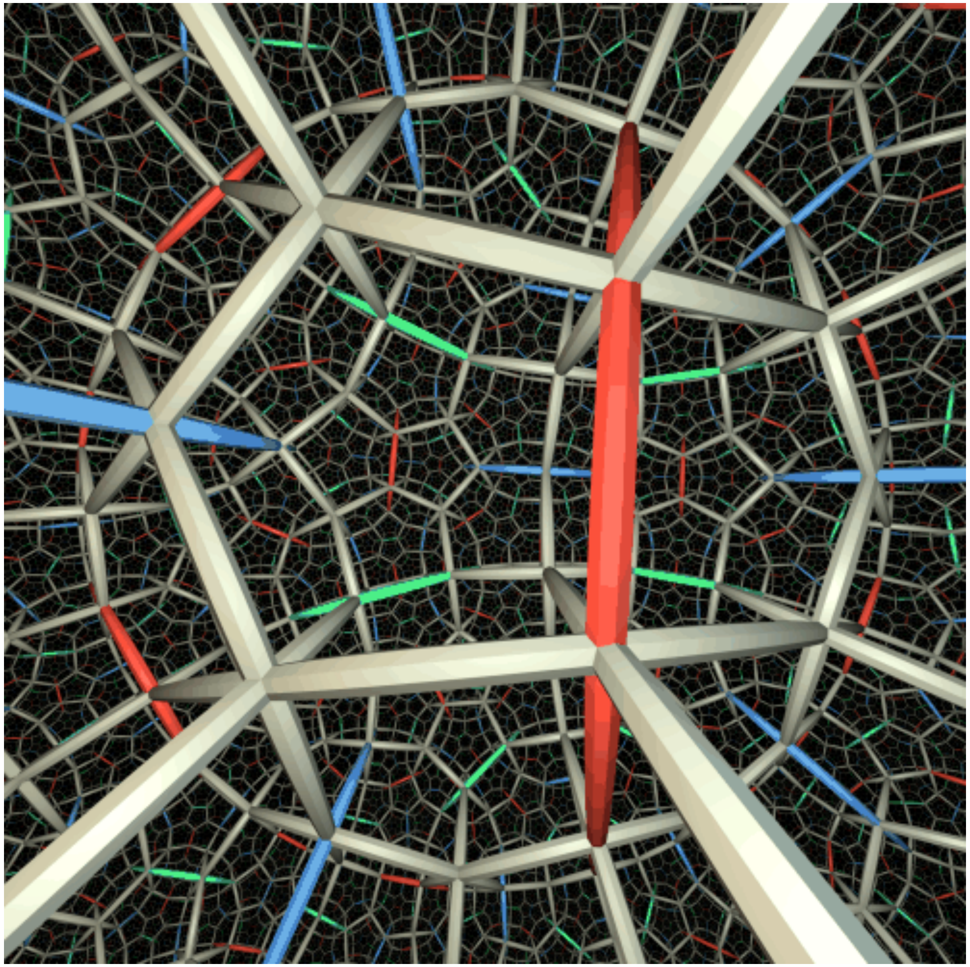


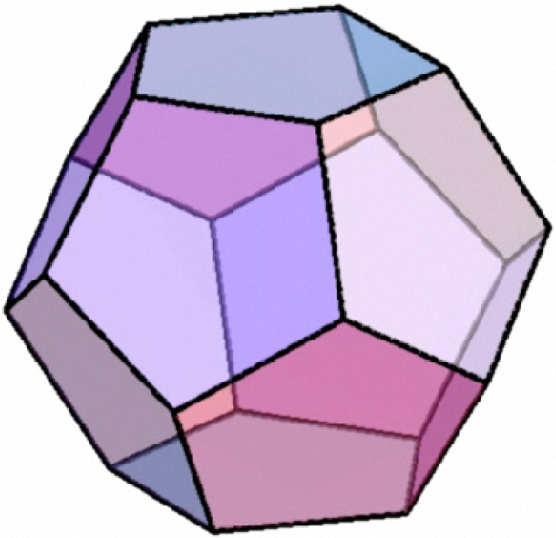
spazi simmetrici

Nil	Sol	$\widetilde{\mathrm{SL}_2 R}$

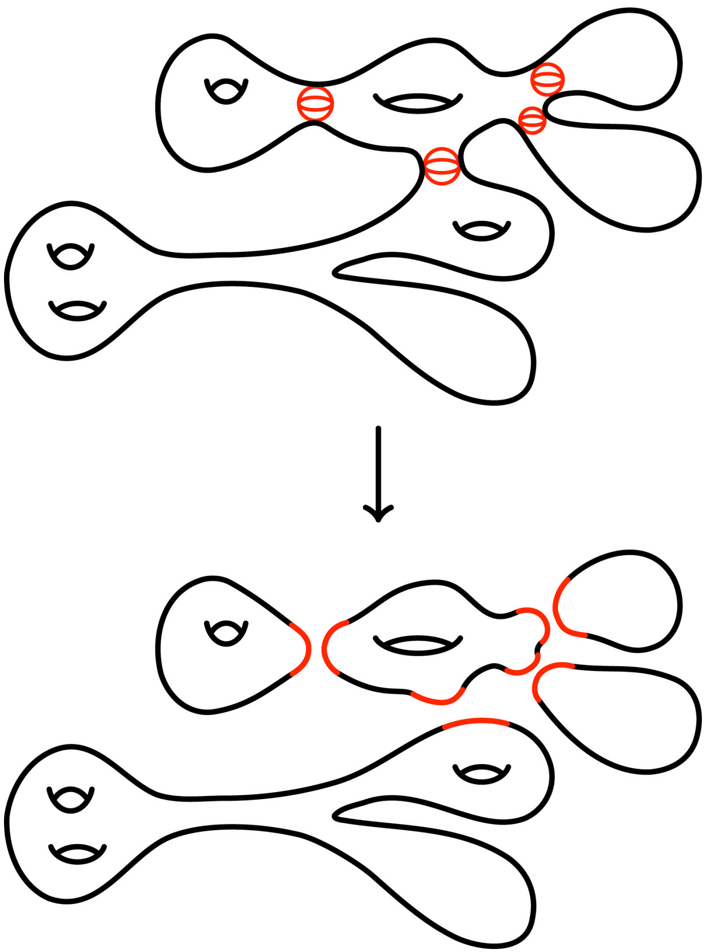
gruppi di Lie

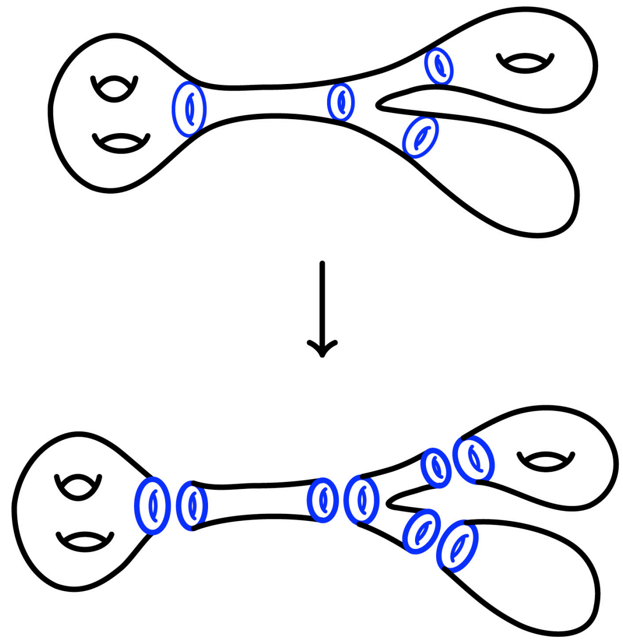
Le geometrie 3-dimensionali





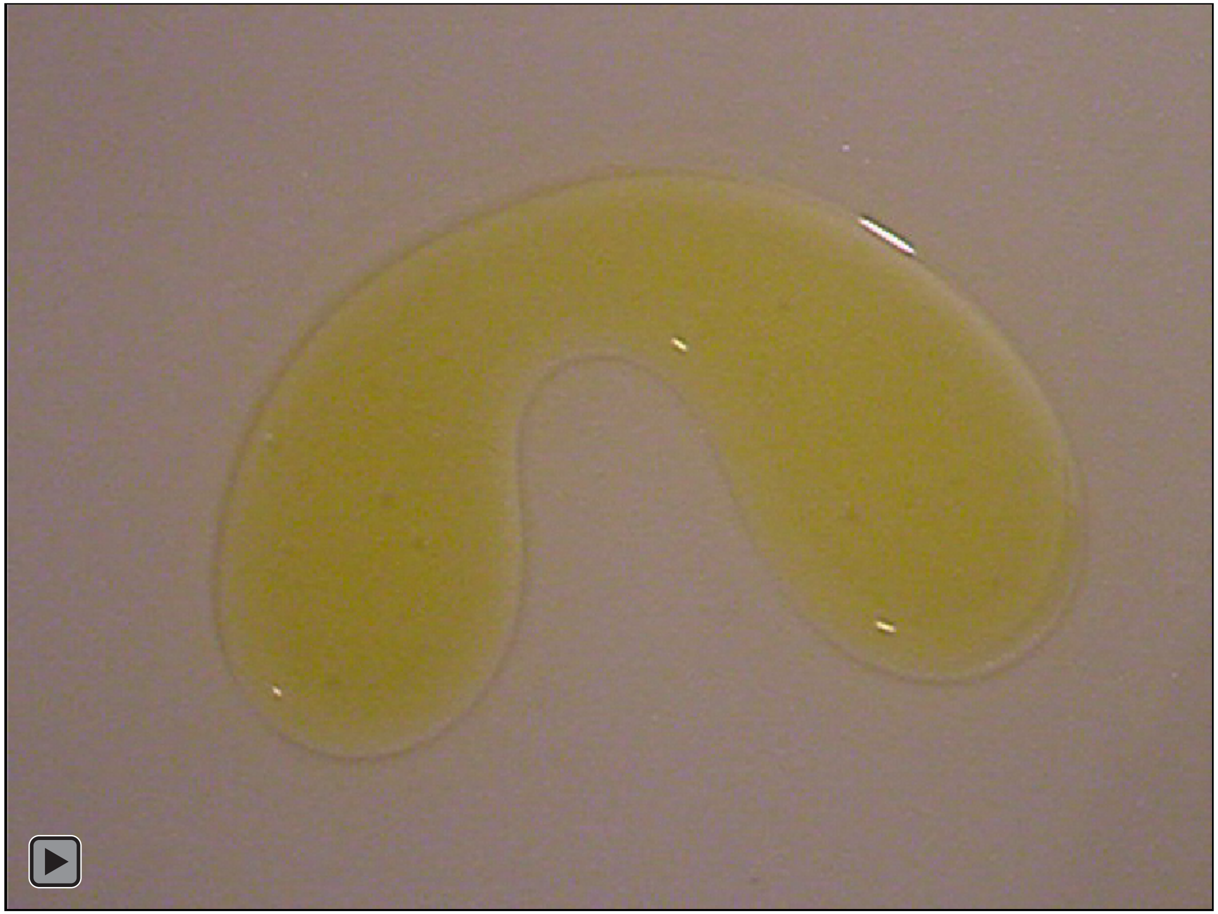
La congettura di Thurston



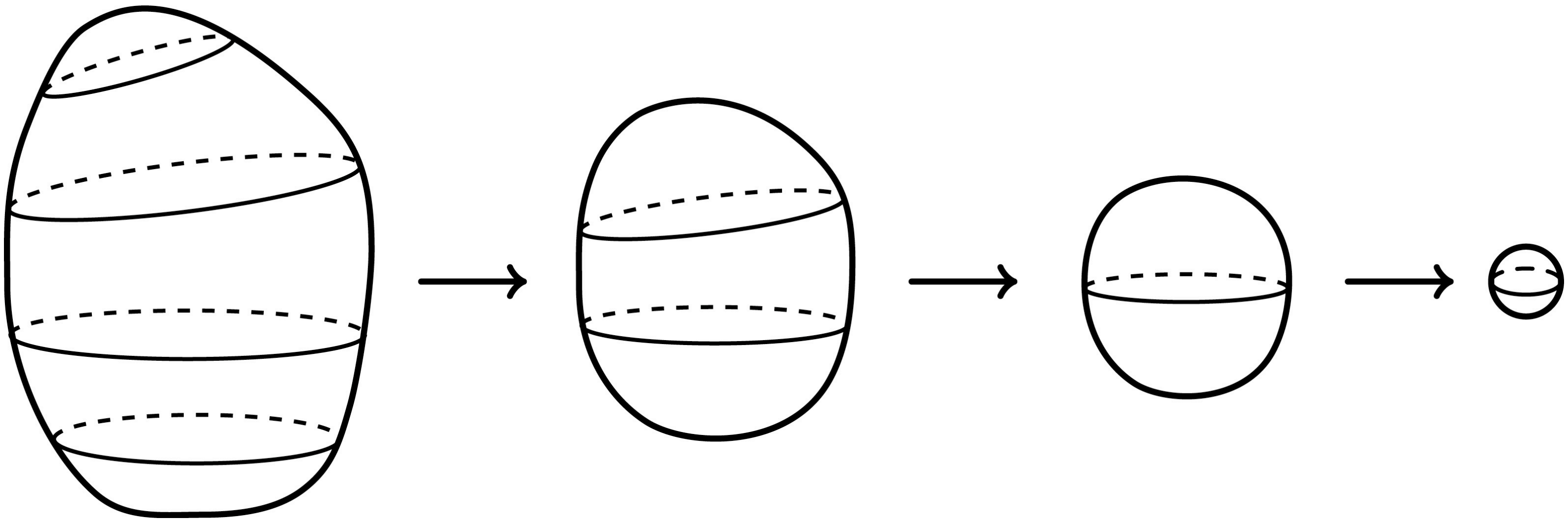


tutti i pezzi sono geometrici

Hamilton: il flusso di Ricci



Hamilton: il flusso di Ricci

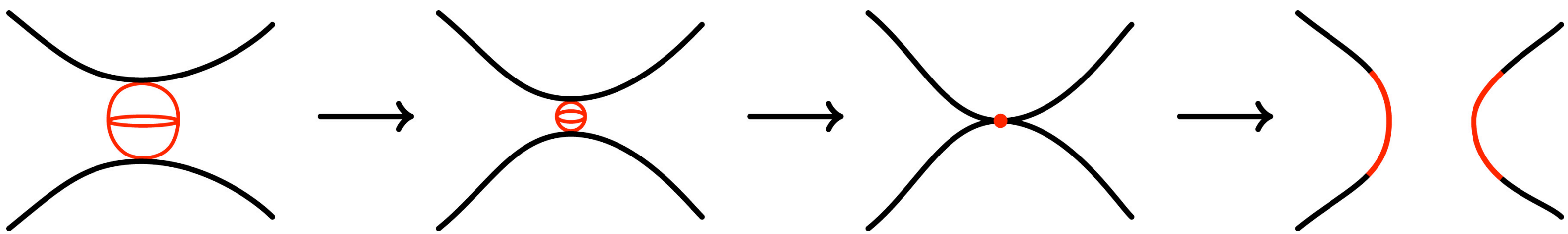


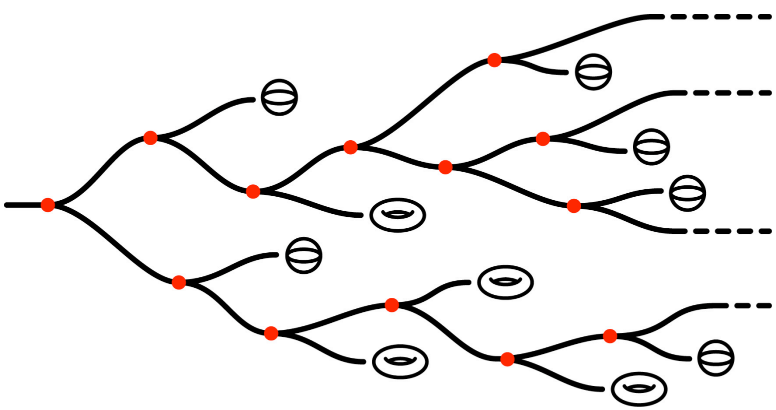
$$g'_{i,j}(t) = -2R_{i,j}(g(t))$$

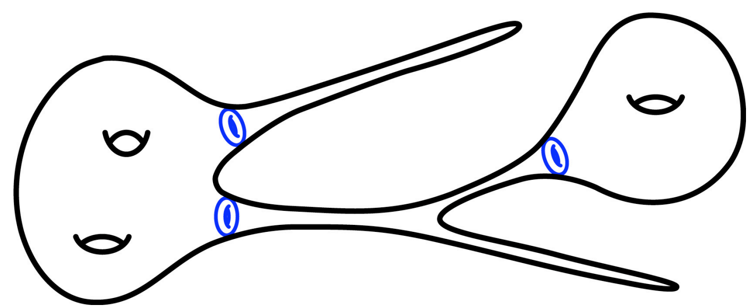
$$R'_{i,j}(t) = \Delta R_{i,j}(g(t)) + Q_{i,j}(g(t))$$

Perelman: le singolarità

Perelman: le singolarità



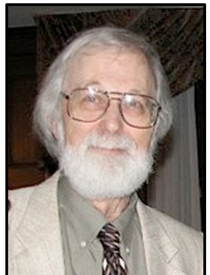


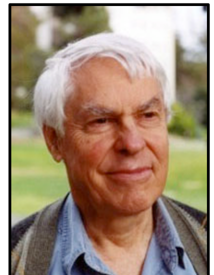


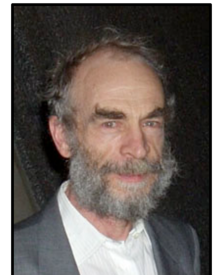
La squadra


Whitehead


Haken


Milnor


Smale


Gromov

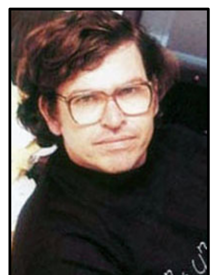

Freedman


Kleiner-Lott

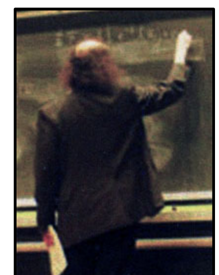

Poincaré

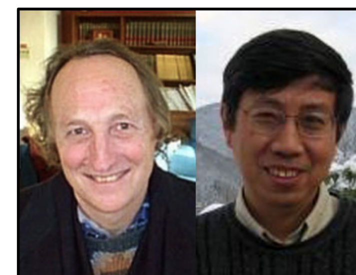

Papa


Moise

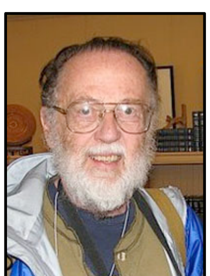

Thurston


Hamilton


Perelman

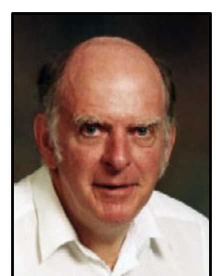

Morgan-Tian


Bing


Stallings


Rourke


Poenaru


Dunwoody


Rubinstein


Cao-Zhu

Qualche lettura

Milnor	**Notices AMS 50 (2003), 1226-1233**
Anderson	**Notices AMS 51 (2004), 184-193**
Perelman	**arXiv:math/0211159 (39 pagine)**
	arXiv:math/0303109 (22 pagine)
	arXiv:math/0307245 (7 pagine)
Kleiner-Lott	**arXiv:math/0605667 (200 pagine)**
Morgan-Tian	**arXiv:math/0607607 (492 pagine)**
Cao-Zhu	**arXiv:math/0612069 (366 pagine)**
D. O'Shea	**La congettura di Poincaré (Rizzoli 2007)**
D. Szpiro	**L'enigma di Poincaré (Apogeo 2008)**